

# Few-Body Aspects of Bose and Fermi Gases

Theoretical frameworks for treating cold few-particle systems: Two-body scattering and hyperspherical perspective

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**Collaborators:**

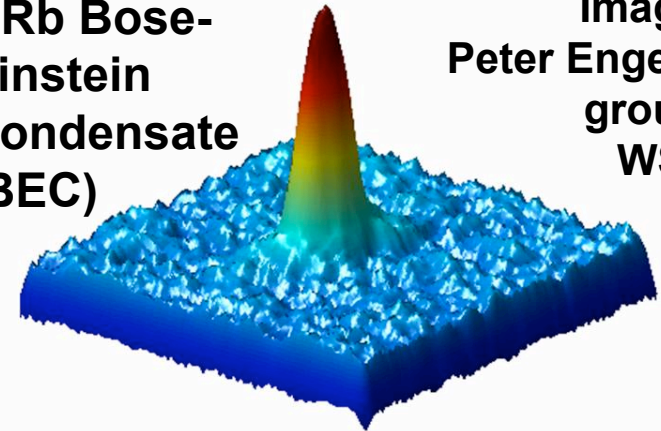
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**At JILA: Chris Greene, Javier von Stecher, Seth Rittenhouse, John Bohn, Shai Ronen, Danielle Bortolotti.**

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**Supported by NSF and ARO.**

**$^{87}\text{Rb}$  Bose-Einstein Condensate (BEC)**



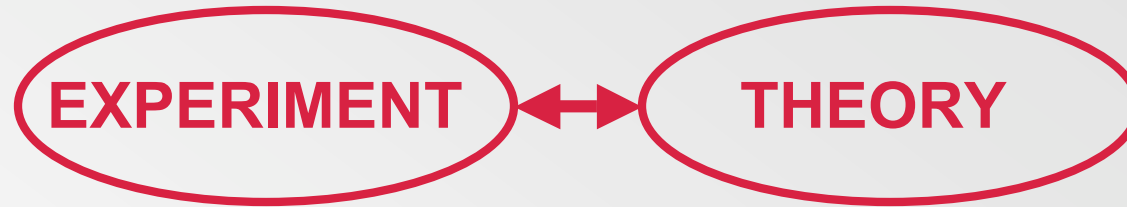
**Image: Peter Engels' group, WSU**

# Outline of This Talk



- **Broad introduction:**
  - **Cold atom physics.**
  - **Few-body physics.**
  - **Overview of lectures.**
  
- **Two-body interactions and scattering length.**
- **Connection between microscopic world and mean-field formulation.**
- **Microscopic understanding of stability of Bose and Fermi gases: Linear Schroedinger equation within hyperspherical framework.**

# The Field of Cold Atom Physics



Nobel Prizes:

Laser cooling (1997):  
Chu, Cohen-Tannoudji, Phillips.

Bose-Einstein  
condensation (2001):  
Cornell, Ketterle, Wieman.

Theory of superconductors  
and superfluids (2003):  
Abrikosov, Ginzberg, Leggett.

Quantum optics and frequency  
comb (2005): Glauber, Hall, Hansch.

molecular  
physics

nuclear  
physics

condensed  
matter

atomic  
physics

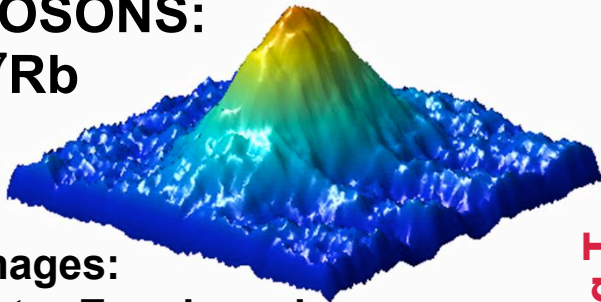
quantum  
information  
science

quantum  
optics

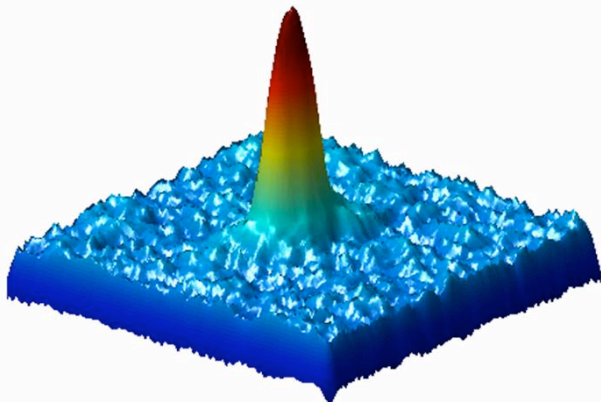
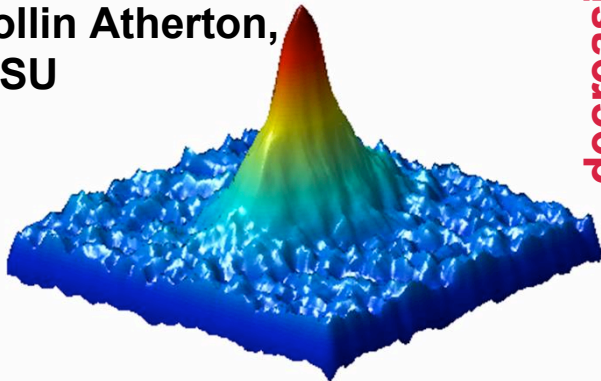
# Reaching Quantum Degeneracy: Bosons versus Fermions

**BOSONS:**

$^{87}\text{Rb}$

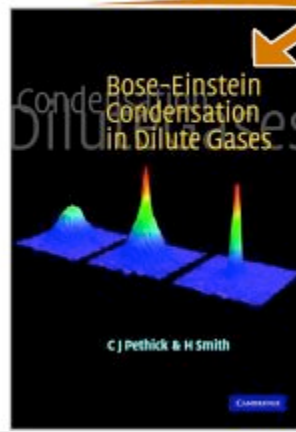


Images:  
Peter Engels and  
Collin Atherton,  
WSU



decreasing T

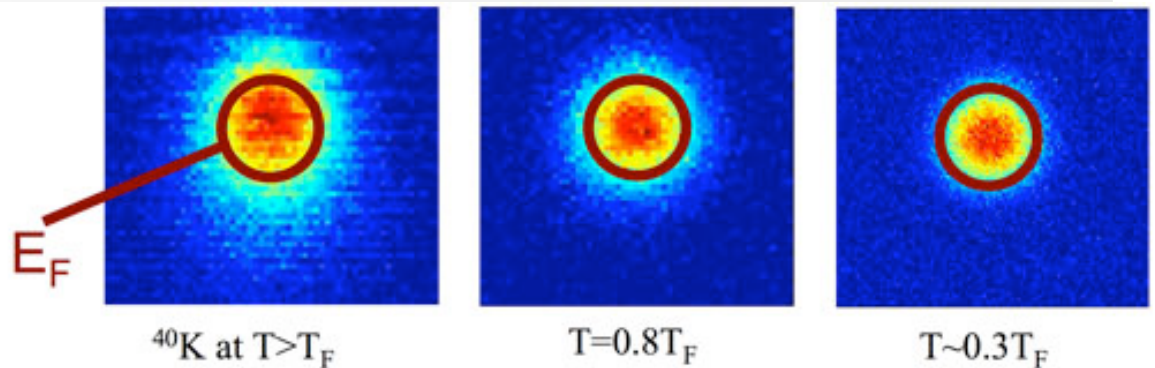
SEARCH INSIDE!™



Transition temperature  
of bosons: a few 100nK  
(BEC: the coldest place  
in the known Universe)

**FERMIONS:**

decreasing T



Images: Thywissen group, University Toronto.

# Composite Atomic Bosons versus Fermions.

- **Boson:** Integer spin; e.g., photon, mesons (q, anti-q).
- **Fermion:** Half-integer spin; e.g., electron, quarks, proton, neutron, baryons (q,q,q).
- **Atoms:** Composite bosons and fermions.
- **E.g.:**  $^4\text{He}$  (B) and  $^3\text{He}$  (F).

$^6\text{Li}$  ( $I=1$ ): 3 electrons  
3 neutrons  
3 protons

Composite fermion

In contrast:

$^7\text{Li}$  ( $I=3/2$ ) composite boson

 **WebElements:** the periodic table on the world-wide web  
<http://www.webelements.com/>

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18										
1 hydrogen H 1.0079																	2 helium He 4.0026										
3 lithium Li 6.941	4 beryllium Be 9.0122												5 boron B 10.811	6 carbon C 12.011	7 nitrogen N 14.007	8 oxygen O 15.999	9 fluorine F 18.998	10 neon Ne 20.180									
11 sodium Na 22.990	12 magnesium Mg 24.305											13 aluminum Al 26.982	14 silicon Si 28.086	15 phosphorus P 30.974	16 sulfur S 32.065	17 chlorine Cl 35.453	18 argon Ar 39.948										
19 potassium K 39.098	20 calcium Ca 40.078											21 scandium Sc 44.956	22 titanium Ti 47.867	23 vanadium V 50.942	24 chromium Cr 51.996	25 manganese Mn 54.938	26 iron Fe 55.845	27 cobalt Co 58.933	28 nickel Ni 58.693	29 copper Cu 63.546	30 zinc Zn 65.39	31 gallium Ga 69.723	32 germanium Ge 72.61	33 arsenic As 74.922	34 selenium Se 78.96	35 bromine Br 79.904	36 krypton Kr 83.80
37 rubidium Rb 85.468	38 strontium Sr 87.62											39 yttrium Y 88.906	40 zirconium Zr 91.224	41 niobium Nb 92.906	42 molybdenum Mo 95.94	43 technetium Tc [98]	44 ruthenium Ru 101.07	45 rhodium Rh 102.91	46 palladium Pd 106.42	47 silver Ag 107.87	48 cadmium Cd 112.41	49 indium In 114.82	50 tin Sn 118.71	51 antimony Sb 121.76	52 tellurium Te 127.60	53 iodine I 126.90	54 xenon Xe 131.29
55 cesium Cs 132.91	56 barium Ba 137.33	57-70 * lanthanoids										71 lutetium Lu 174.97	72 hafnium Hf 178.49	73 tantalum Ta 180.95	74 tungsten W 183.84	75 rhenium Re 186.21	76 osmium Os 190.23	77 iridium Ir 192.22	78 platinum Pt 195.08	79 gold Au 196.97	80 mercury Hg 200.59	81 thallium Tl 204.38	82 lead Pb 207.2	83 bismuth Bi 208.98	84 polonium Po [209]	85 astatine At [210]	86 radon Rn [222]
87 francium Fr [223]	88 radium Ra [226]	89-102 ** actinoids										103 lawrencium Lr [262]	104 rutherfordium Rf [261]	105 dubnium Db [262]	106 seaborgium Sg [263]	107 bohrium Bh [264]	108 hassium Hs [265]	109 meitnerium Mt [266]	110 darmstadtium Ds [271]	111 roentgenium Rg [272]	112 copernicium Cn [277]	113 nihonium Nh [285]	114 flerovium Fl [289]	115 moscovium Mc [288]	116 livermorium Lv [293]	117 tennessine Ts [294]	118 oganeson Og [294]

lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	esboium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

Symbols and names: the symbols and names of the elements, and their spellings are those recommended by the International Union of Pure and Applied Chemistry (IUPAC) - <http://www.iupac.org/>. Names have yet to be proposed for the most recently discovered elements 111-112 and 114 so those used here are IUPAC's temporary systematic names. In the USA and some other countries, the spellings **aluminium** and **caesium** are normal while in the UK and elsewhere the common spelling is **aluminum**. Group labels: the numeric systems (1-10) used here is the current IUPAC convention. Atomic weights (mean relative masses): Apart from the heaviest elements, these are the IUPAC 2001 values and given to 5 significant figures. Elements for which the atomic weight is given within square brackets have no stable isotopes and are represented by the element's longest-lived isotope. ©2003 Dr Mark J Winter (WebElements Ltd and University of Sheffield, [webelements@sheffield.ac.uk](http://www.webelements.com/)). All rights reserved. For updates to this table see <http://www.webelements.com/webelements/support/mediawiki/>. Version date: 17 March 2003.

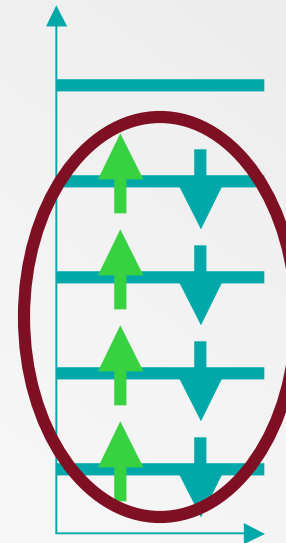
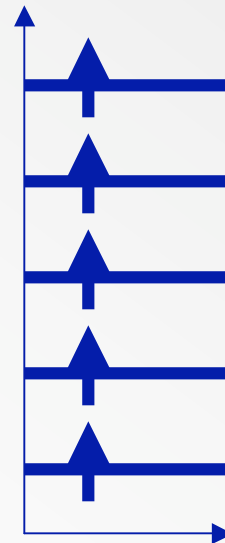
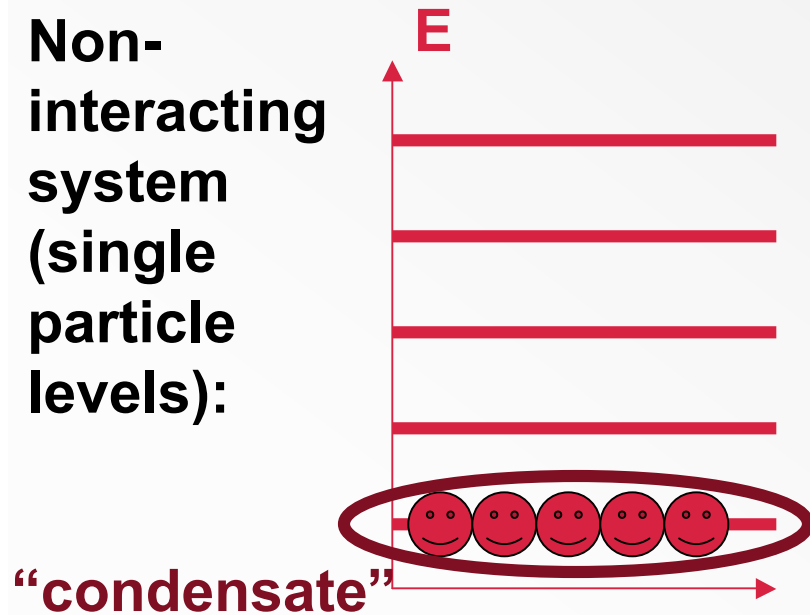


# Bose versus Fermi Statistics: Gas at Low Temperature

One-component Bose gas: 

One-component spin-polarized Fermi gas: 

Two-component Fermi gas: 

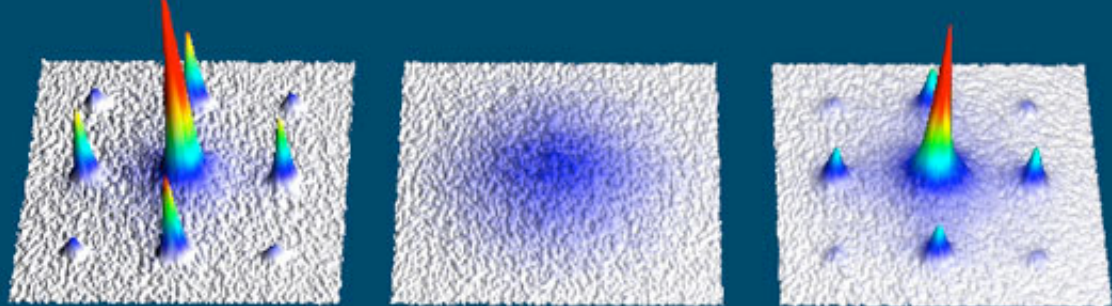


Quantum degenerate Fermi gas

# Selected Highlights from Cold Atom Experiments

MPI, Hansch group (2002): Quantum phase transition:

Phase coherence    No phase coh.

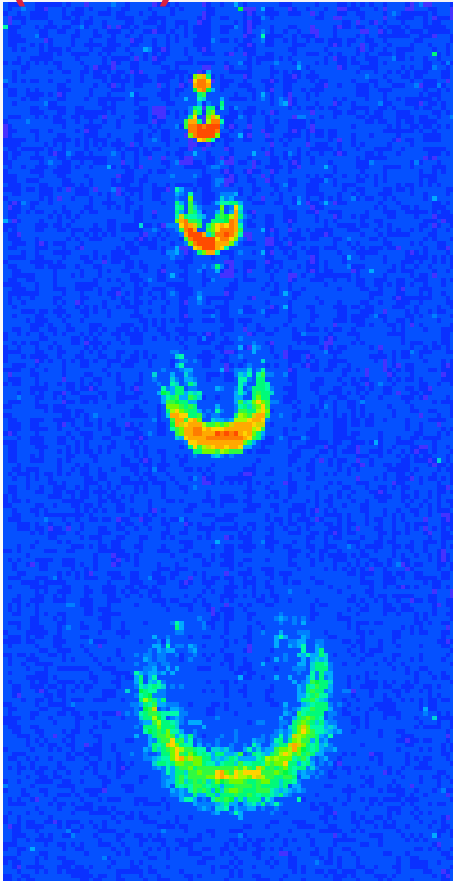


Superfluid

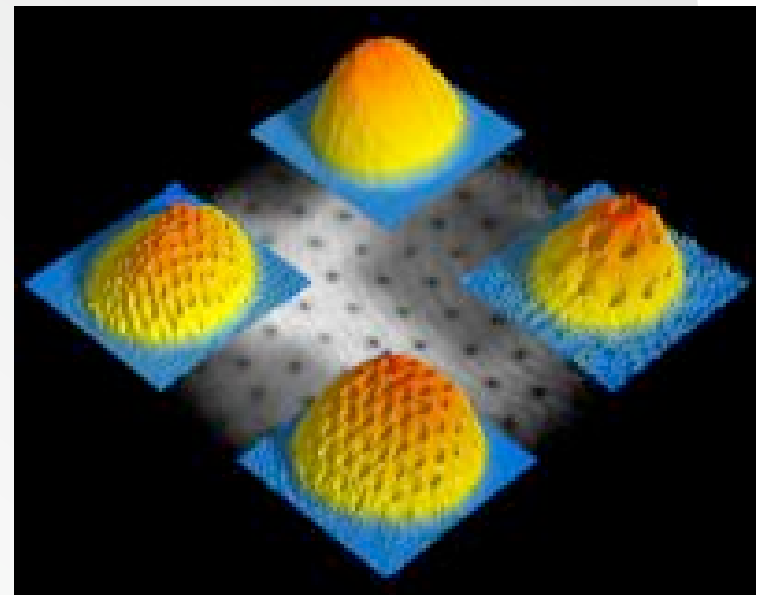
Mott insulator

Superfluid

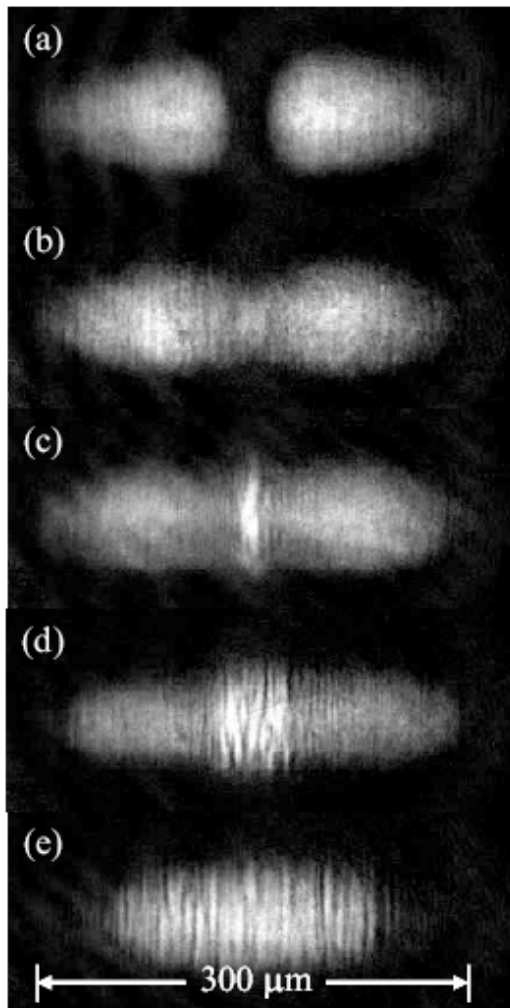
MIT, Ketterle group (1997): Atom laser.



MIT, Ketterle group (2001): Vortex lattice in Na BEC.



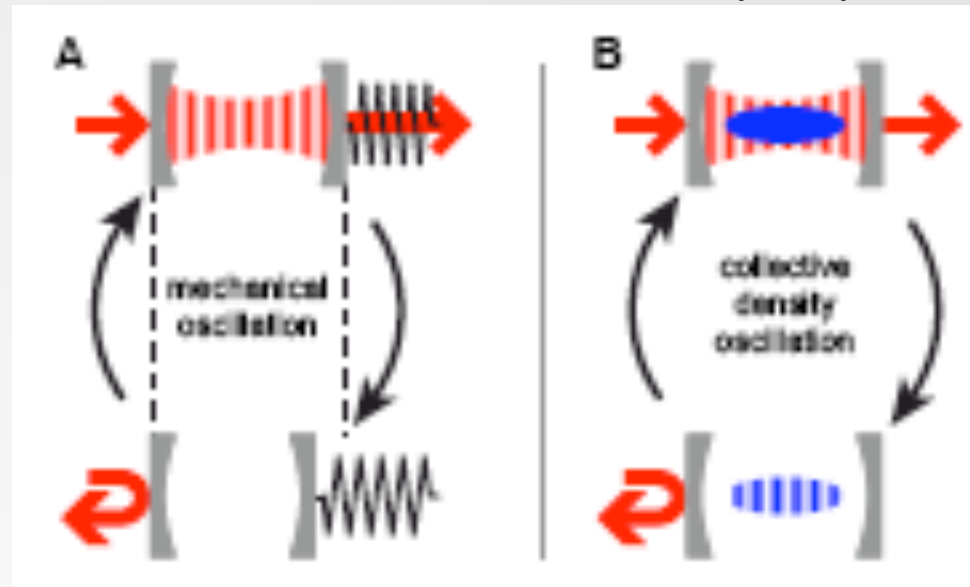
# More Highlights from Cold Atom Experiments...



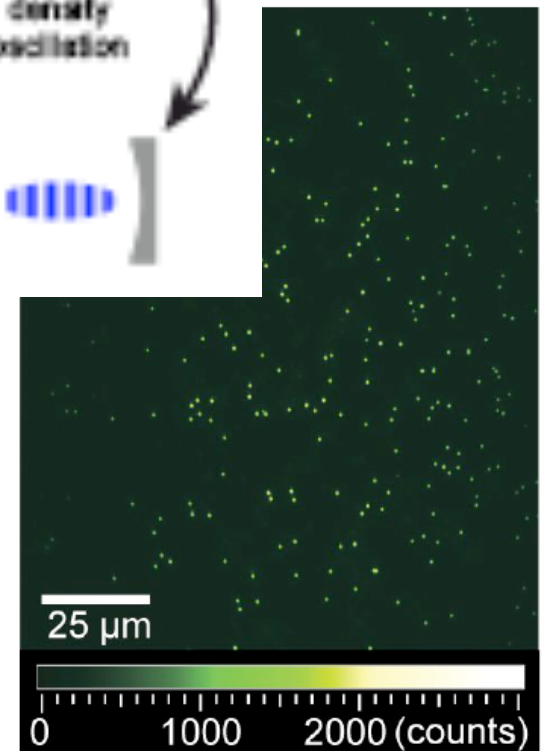
**Quantum shock**  
Chang et al., PRL (2008)

## Cavity opto-mechanics with a BEC

Brennecke et al., Science 322, 238 (2008)



**Quantum gas microscope**  
Bakr et al.,  
arXiv:0908.01744.





# Few-Body Highlights: Optical Lattice

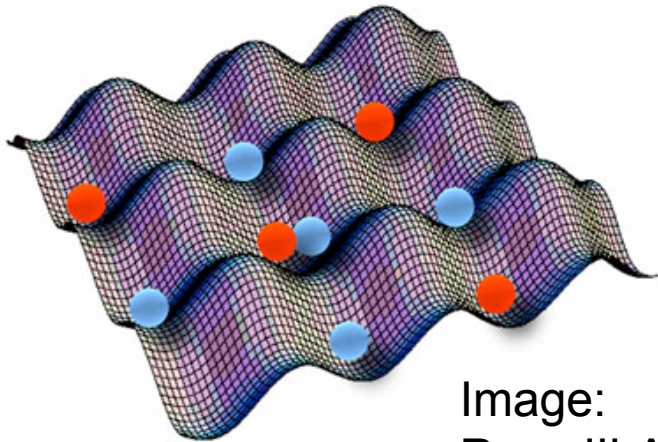
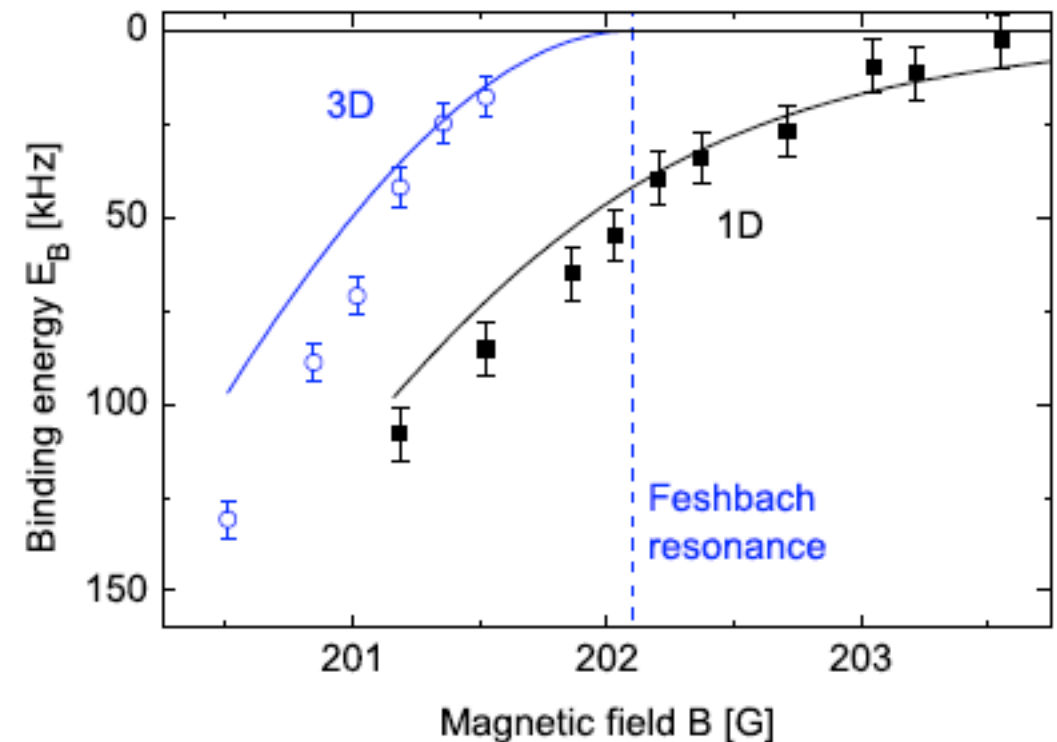


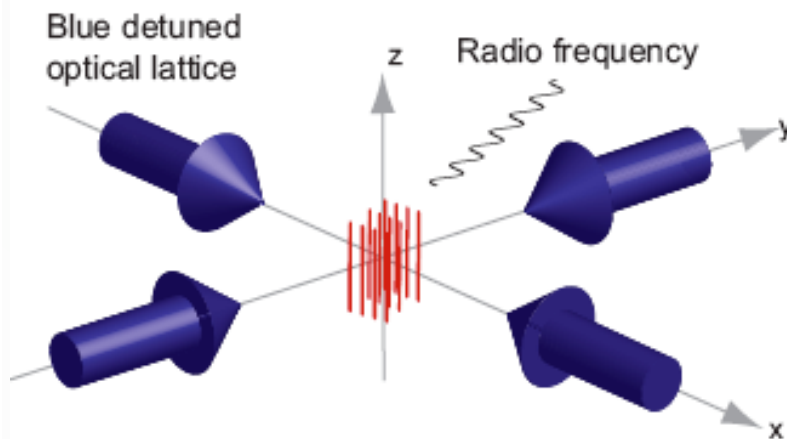
Image:  
Rey, JILA

## Measurement of two-body binding energy in 3D and 1D

Moritz et al., PRL 94, 210401 (2005)



## Designing effectively 1D and 2D confinement:

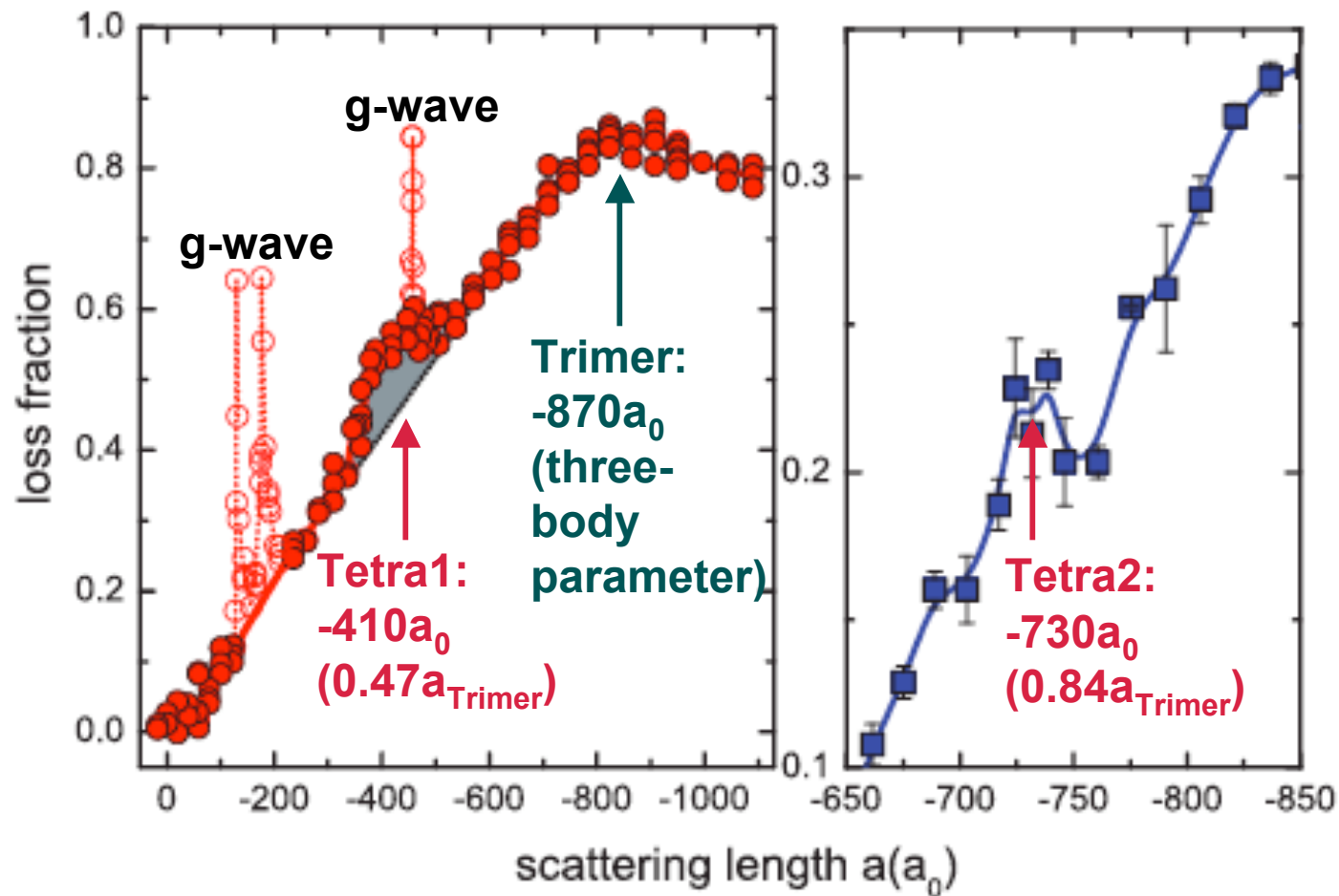


Palzer et al., PRL 103, 150601 (2009)

# Measurement of Loss Rate for Non-Degenerate Bosonic $^{133}\text{Cs}$ Sample

First measurement of universal 4-body physics (probe of Efimov physics)

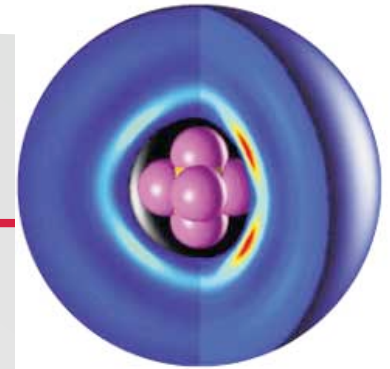
Ferlaino et al., PRL 102, 140401 (2009).



How to stabilize these delicate trimers and tetramers?

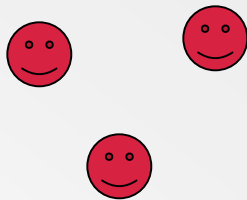
# Few-Body Physics: Bridge to Macroscopic World

Toennies  
et al.,  
Physics  
Today 54,  
31 (2001).

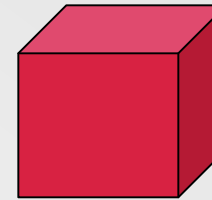


- Microscopic to macroscopic:

atomic/  
molecular



→  
"mesoscopic"



condensed  
matter

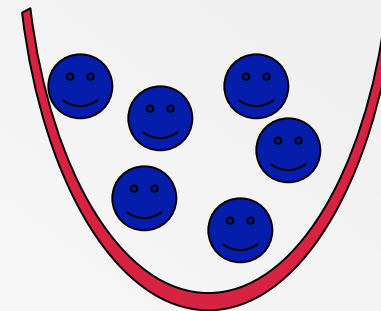
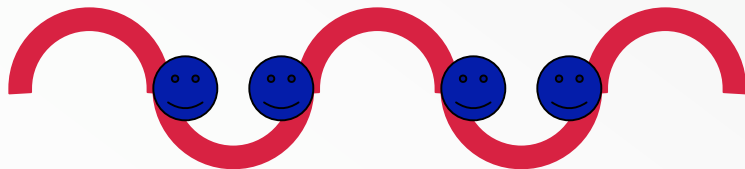
- Other examples:

- Doped helium clusters: Molecular rotations, microscopic superfluidity,...
- Metal clusters: conductivity, designing materials,...

- What is special about cold atomic Bose and Fermi systems?

- Universal behavior.
- Much experimental progress!

optical  
lattice



External  
confining  
potential

# Topics to be Covered

---

equal-mass  
Fermi gas

unequal-mass  
Fermi gas

Bose  
gas

dipolar  
Bose gas

Scattering  
theory

Monte Carlo  
techniques

hyperspherical  
framework

Stochastic  
variational approach

Virial  
expansion

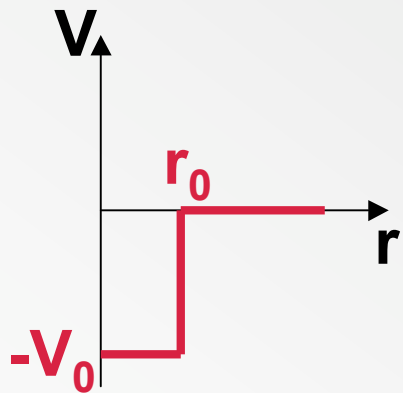
Mean-field  
theory

# Generalized Scattering Length: Central Two-Body Interaction $V(r)$

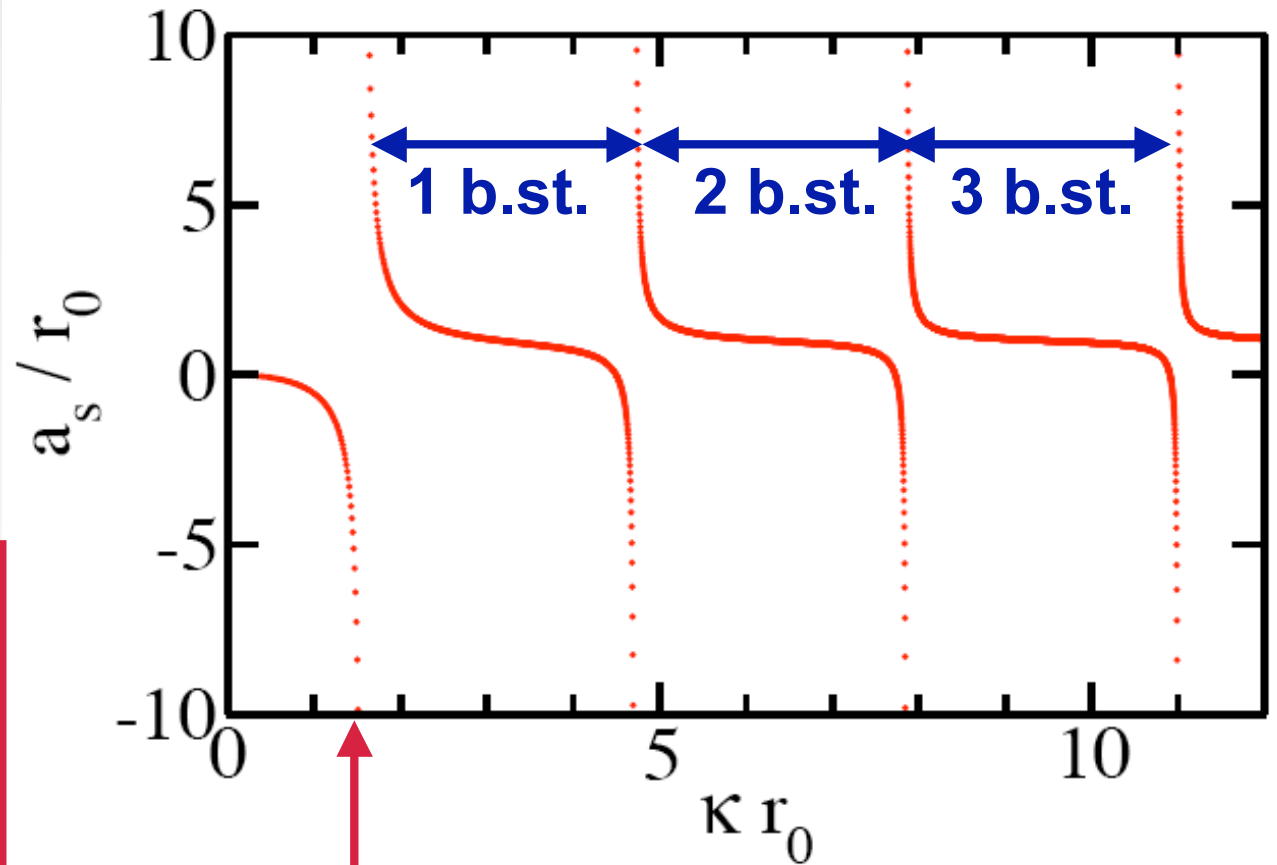
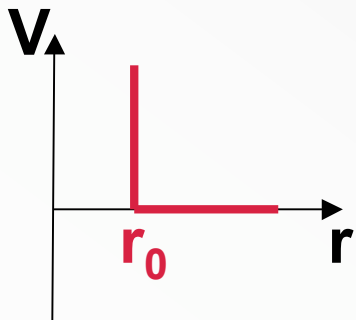
- **Two-particle system: Separate off CM degrees of freedom.**
- **Partial wave decomposition of wave function in relative coordinates:  $\psi(r, \theta, \varphi) = \sum_{lm} R_l(kr) Y_{lm}(\theta, \varphi)$ .**
- **Scaling:  $u_l(kr) = kr R_l(kr)$ .**
- **Outside: Radial solution defines phase shifts  $\delta_l(k)$ .  
 $R_l(kr) = A_l(k)[j_l(kr) - \tan(\delta_l(k)) n_l(kr)]$ .**
- **Generalized energy-dependent scattering lengths (SR)  
 $a_l(k) = -\tan(\delta_l(k)) / k^{2l+1}$ .**
- **Generalized energy-independent scattering lengths  
 $a_l = \lim_{k \rightarrow 0} a_l(k)$ .**
- **Inside solution: Same as bound state solution.**



# Example: Square-Well Interaction Potential



In contrast:  
Hardcore potential:  
 $a_s = r_0$

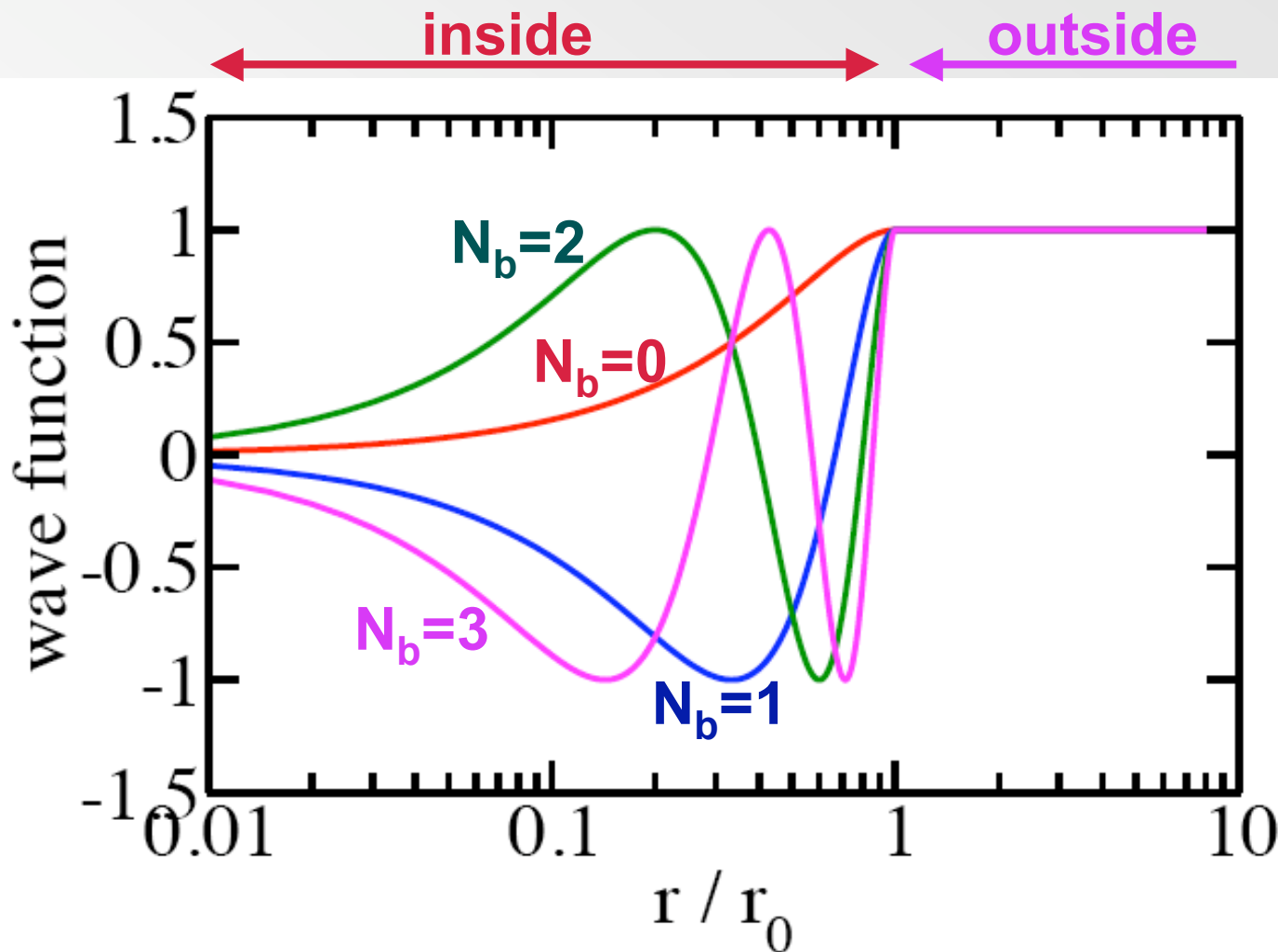


Infinite scattering length (unitarity).  
Small range.

$$\kappa \propto \sqrt{V_0}$$

increasing well depth  
(attraction)

# Zero-Energy Scattering Wave Function at Unitarity



Outside solution identical for all four potential depths.

Outside ( $a_s \rightarrow \infty$ ):

$$u_s(r) \propto$$

$$\sin(kr) +$$

$$\tan(\delta_s(k)) \cos(kr) \propto$$

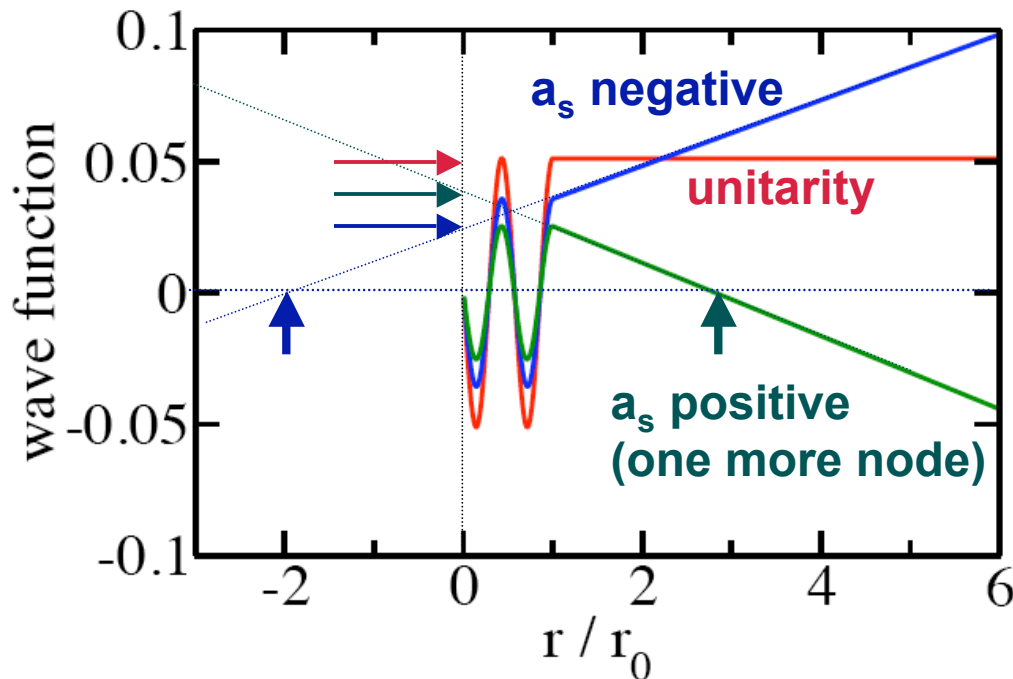
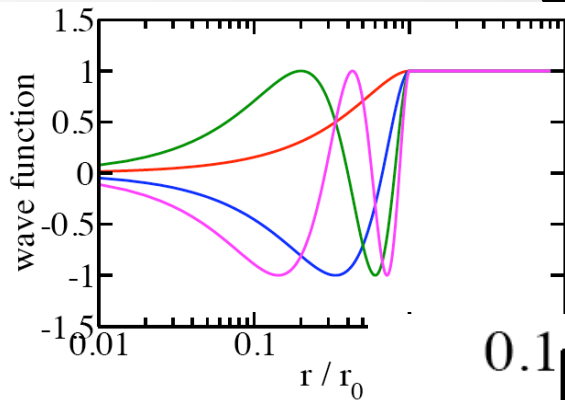
$$kr + \tan(\delta_s(k)) \propto$$

$$r - a_s \propto r/a_s - 1 \propto$$

const

Inside solution depends on details of interaction potential.

# Zero-Energy Scattering Wave Function



Outside:

$$u_s(r) \propto$$

$$\sin(kr) +$$

$$\tan(\delta_s(k)) \cos(kr) \propto$$

$$kr + \tan(\delta_s(k)) \propto$$

$$r - a_s$$

Inside solution depends on details of interaction potential.

These details are not being probed at low temperature because...

# deBroglie Wave Length: Degeneracy and Resolution

$$\lambda = h / (2\pi m k_B T)^{1/2}$$

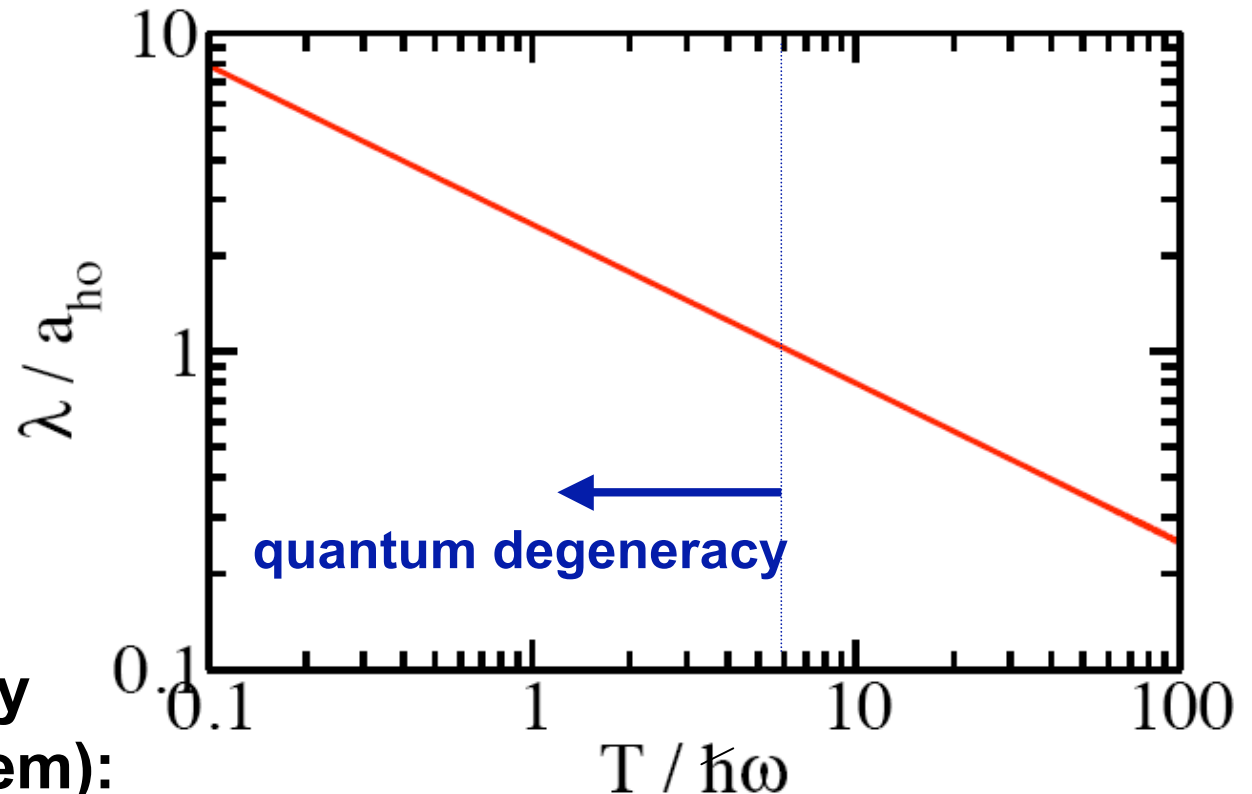
Assuming a trapped system with osc. length  $a_{ho}$ :

$$\lambda / a_{ho} = (2\pi \hbar \nu / k_B T)^{1/2}$$

Quantum degeneracy (homogeneous system):

$$\lambda / \langle r_{ij} \rangle > (2.6)^{1/3} \approx 1.38$$

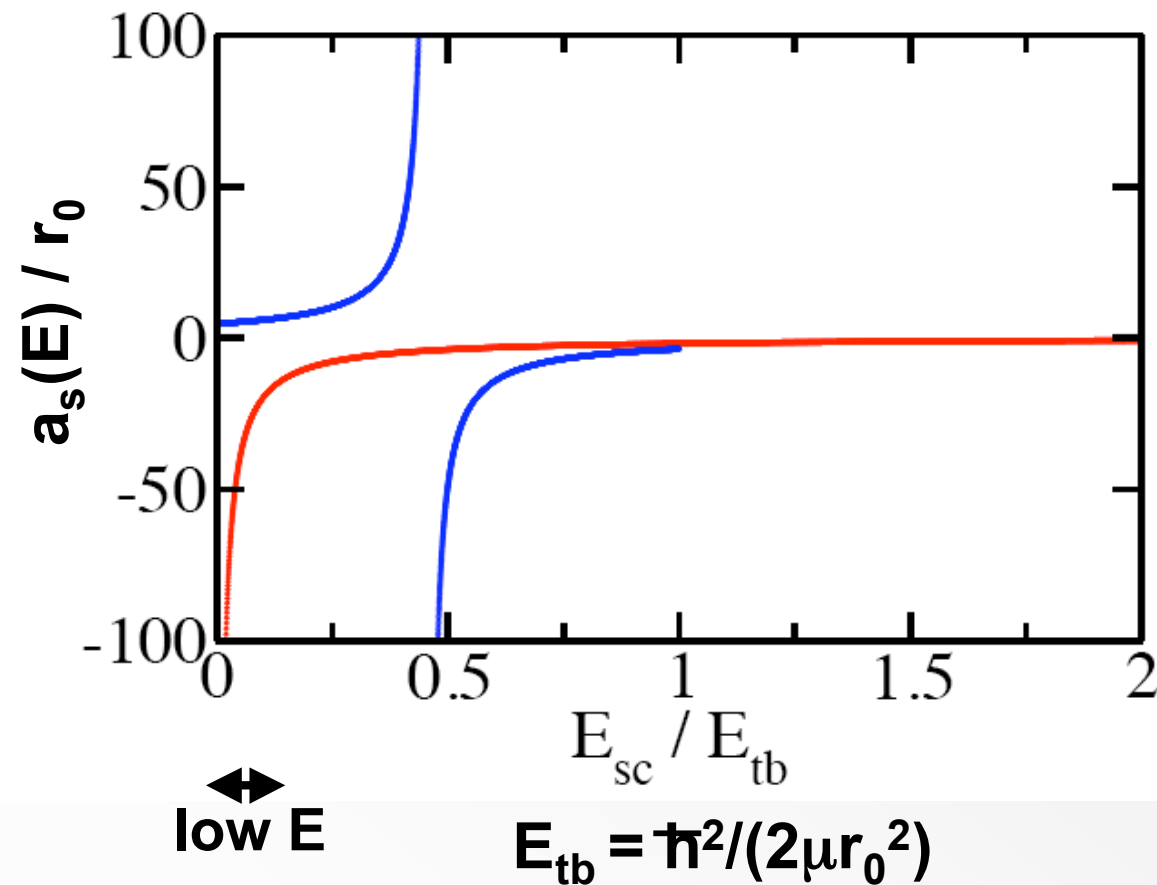
Resolution is set by  $\lambda$  ( $r_0 \ll a_{ho}, \lambda$ ).



Low T: Collisions are too slow to probe small r piece (high momentum piece) of wave fct.

High T: Can probe small r piece of wave fct.

# Temperature Determines Collision Energy



Low energy physics implies:

$$E_{sc} \ll E_{tb}$$

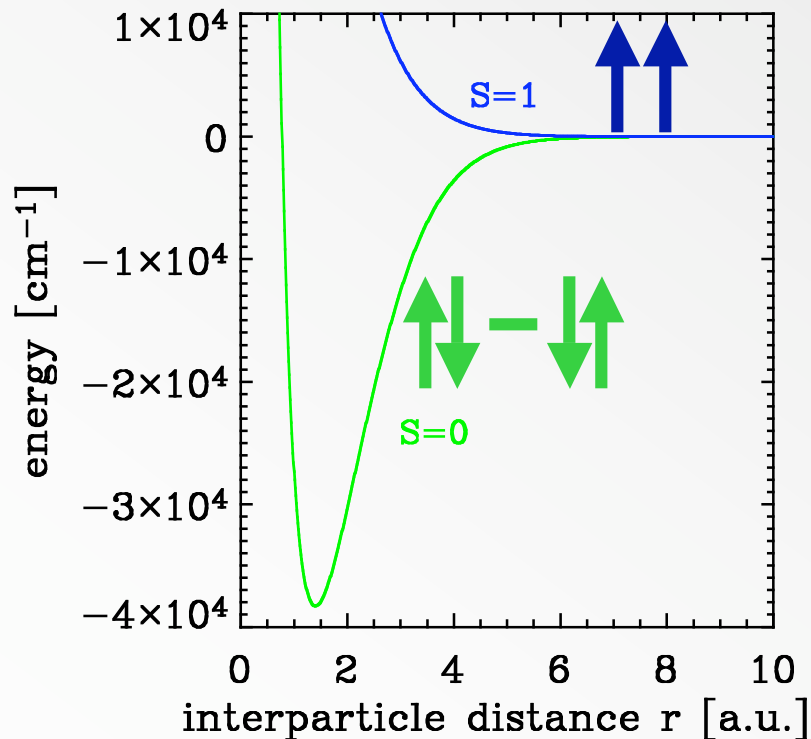
$E_{bound} \ll E_{tb}$   
(dimer size larger than range  $r_0$ )

Weak binding:  
 $E_{bound} \approx -\hbar^2/ma_s^2$

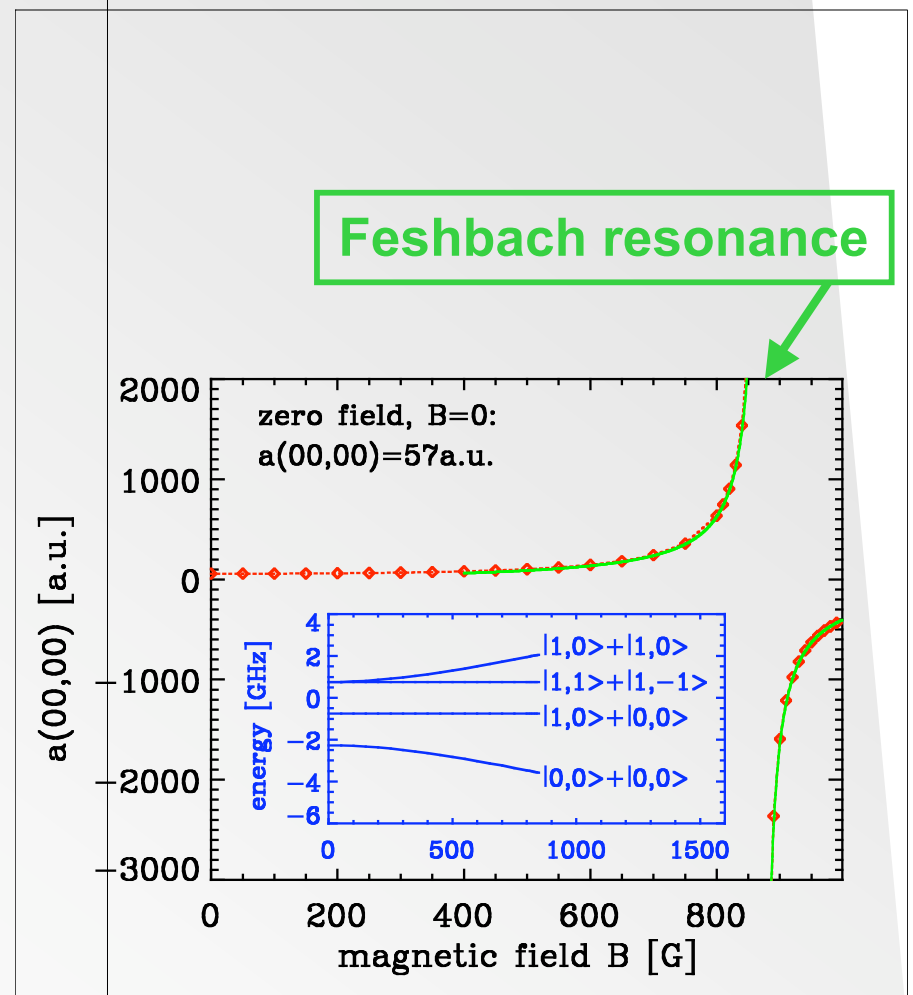


# But: Scattering Length Determined by Coupled Channel Interaction

- Hyperfine Hamiltonian couples singlet and triplet potential curves.
- In the vicinity of Fano-Feshbach resonance, scattering length tunable (here, tritium-tritium system).

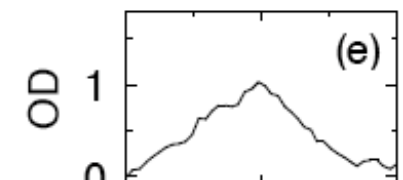
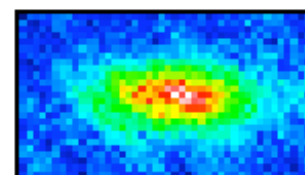
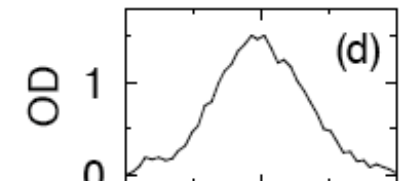
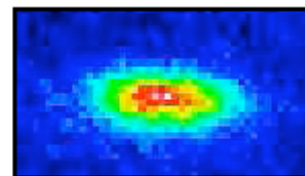
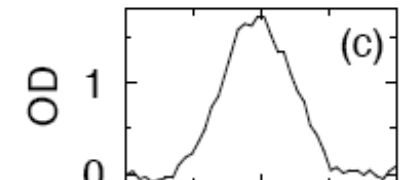
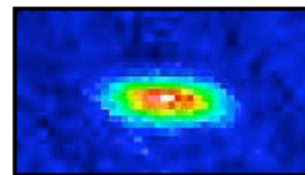
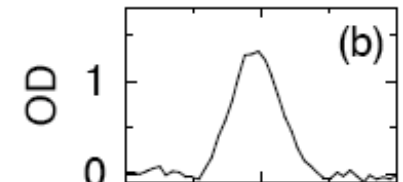
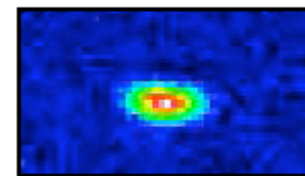
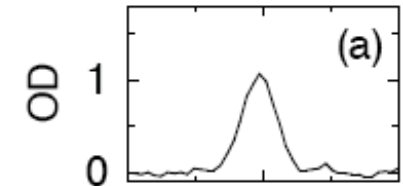
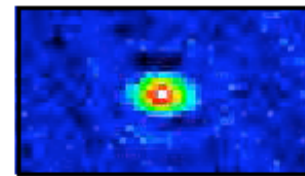
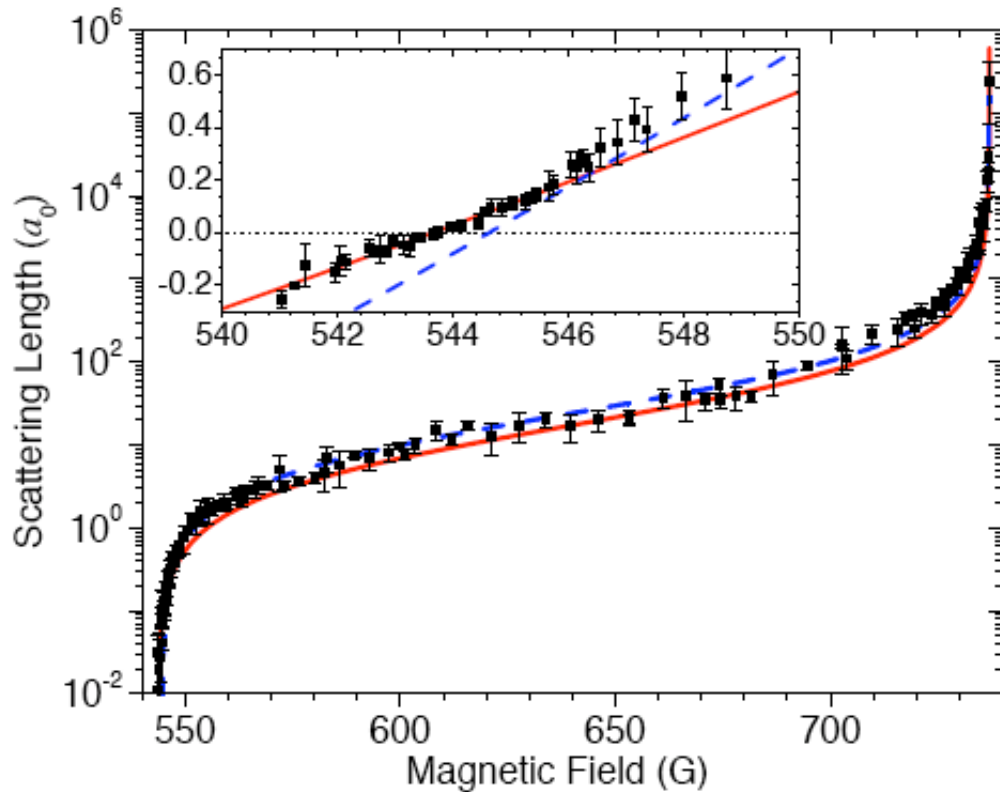


See Blume et al., PRL 89, 163402 (2002).



# Experimental Feshbach Resonance Tuning

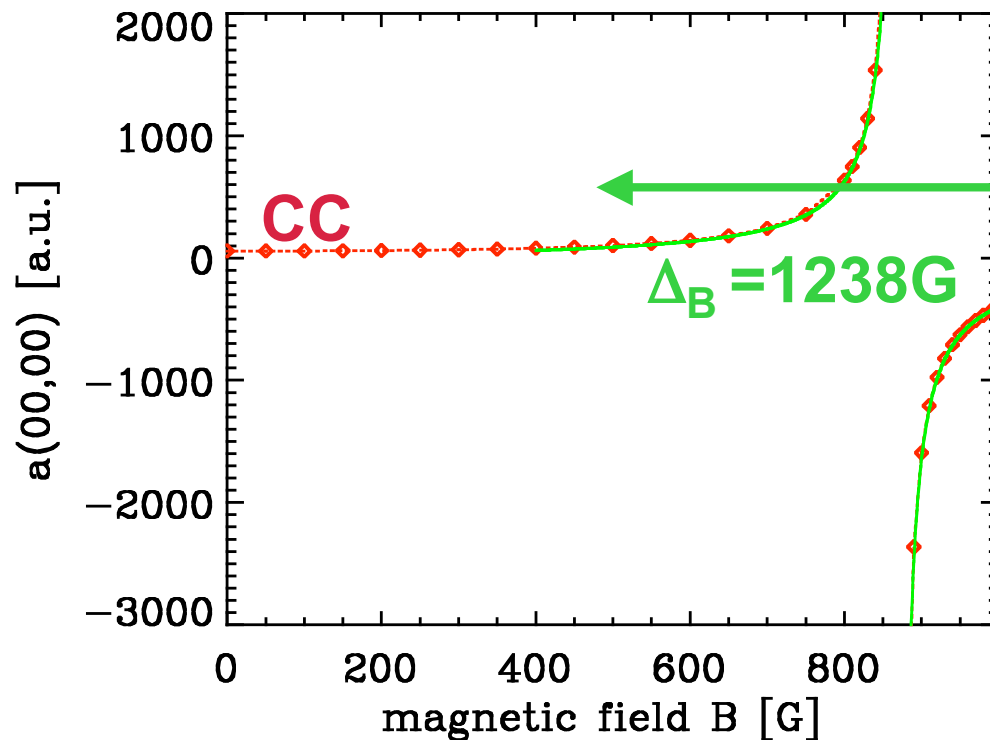
Extreme tunability of  $^7\text{Li}$  Bose gas  
Pollack et al., PRL 102, 090402 (2009)



Roberts et al.,  
PRL (2004)

0 50 100  
Horizontal Position ( $\mu\text{m}$ )

# Broad and Narrow Resonance: Can We Get Away With Single Channel Model?



$$a_{\text{eff}} = a_{\text{bg}}(1 + \Delta_B / (B - B_R))$$

$$\Delta_B \sim \Delta_E$$

**Broad resonance:**

$$E_{\text{char}} \gg \Delta_E \text{ or } r^* k_{\text{char}} \ll 1$$

$$r^* = \hbar^2 / (m a_{\text{bg}} \Delta_B \mu_{\text{oc}})$$

$$\text{e.g., } ^{40}\text{K: } r^* \sim 10 \text{ \AA}$$

**Fermi gas in strongly-interacting regime:**

$$k_{\text{char}} = k_F \text{ (negligible occupation of closed channel molecule)}$$

**Composite molecular Bose gas:**

$$E_{\text{char}} = E_{\text{bind}}, k_{\text{char}} = 1/a_s$$

# Physics Determined by s-Wave Scattering Length

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- Reaching quantum degeneracy requires thermalization.
- Efficient thermalization requires finite  $a_s$ .
- No thermalization for single-component Fermi gas.
- Three examples:
  - 1. Gas-like states of trapped two-particle system are determined by s-wave scattering length  $a_s$ .
  - 2. Stability/instability of Bose gas determined by scattering length.
  - 3. Stability/instability of multi-component Fermi gas determined by scattering length plus Fermi pressure (Pauli exclusion principle).

# 1. Two-Particle System: Replace Atom-Atom Interaction by ZR Interaction

- Start with ab initio atom-atom potential.
- Coupled channel calculation provides phase shifts  $\delta_l(k)$ .
- Construct zero-range pseudo-potential with same  $a_s$  (outside solution):

$$V(\vec{r}) = \frac{4\pi \hbar^2 a_s}{m} \delta^{(3)}(\vec{r}) \left[ \frac{\partial}{\partial r} r \right]$$

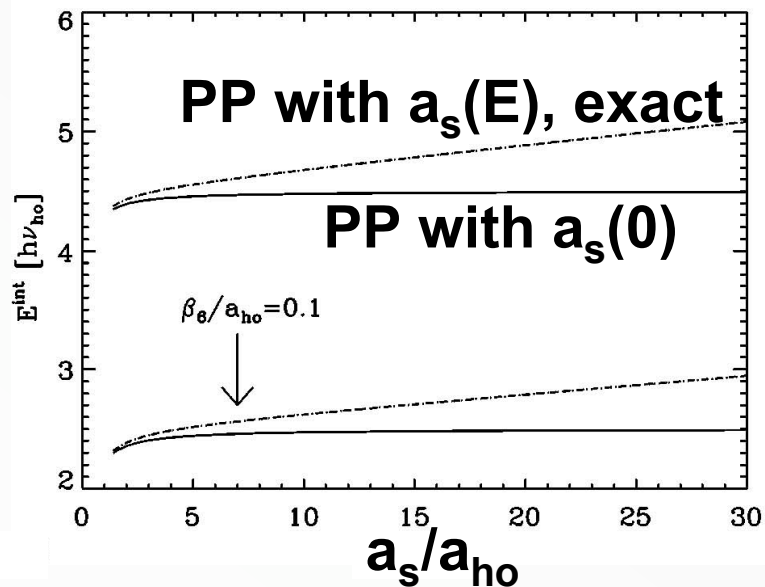
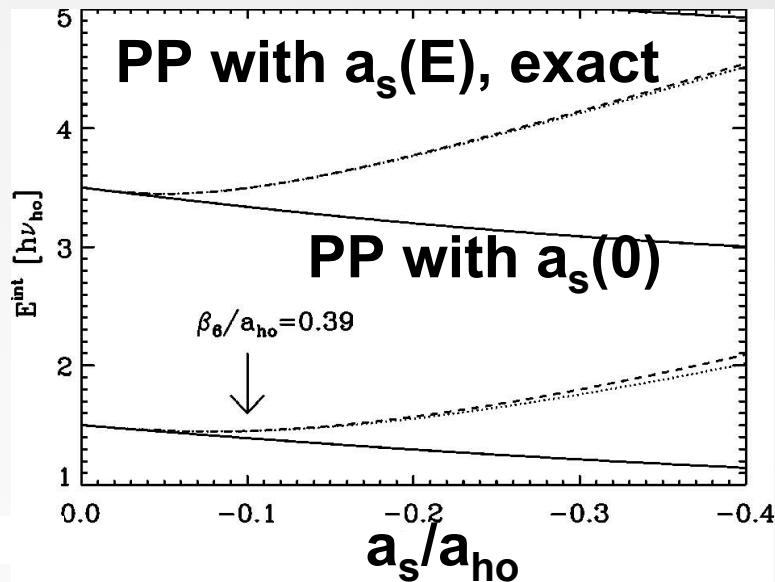
Cures  $1/r$  divergence of radial function (specific to 3D)

Analytical treatments

[Huang and Yang, PR 105, 767 (1957)]



# 1. Two Particles under External Harmonic Confinement



- Two-particle energy spectrum known semi-analytically: Simple transcendental equation [Busch et al., Found. of Physics (1998)].
- Self-consistent solution when  $a_s = a_s(E)$  [Blume and Greene, PRA 65,043613 (2002); see also Bolda et al. (PRA, 2002)].
- Energy-independent pseudo-potential, i.e., use of  $a_s(0)$ , works if  $|a_s| \ll a_{ho}$ .
- Energy-dependent pseudo-potential, i.e., use of  $a_s(E)$ , works if  $r_{vdW} \ll a_{ho}$ .

# 2. Stability of Bose Gas Under Harmonic Confinement

N bosons in a box.  
Constant density.  
Periodic boundary conditions.

**Positive scattering length: Stable gas (not self-bound).**

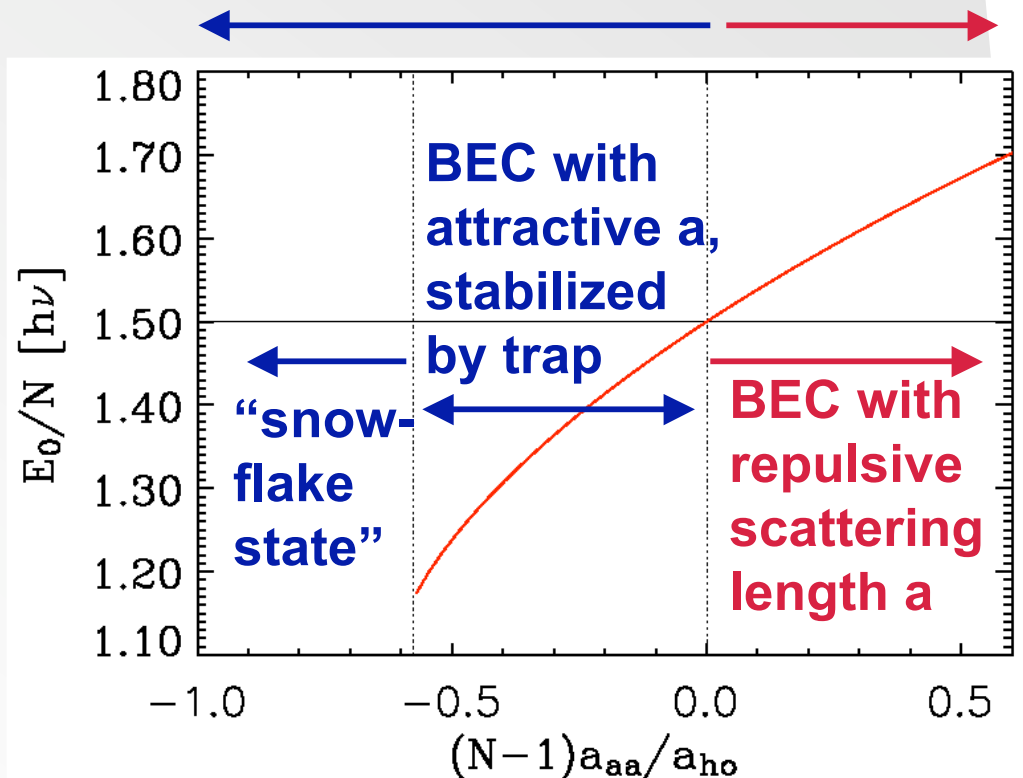
**Negative scattering length: Gas not stable; collapse toward solid or liquid (self-bound).**

Bosonic atoms in harmonic trap  
(mean-field GP treatment):

**Positive scattering length:  
Effectively repulsive  
interaction.**

**Negative scattering length:  
Effectively attractive  
interaction.**

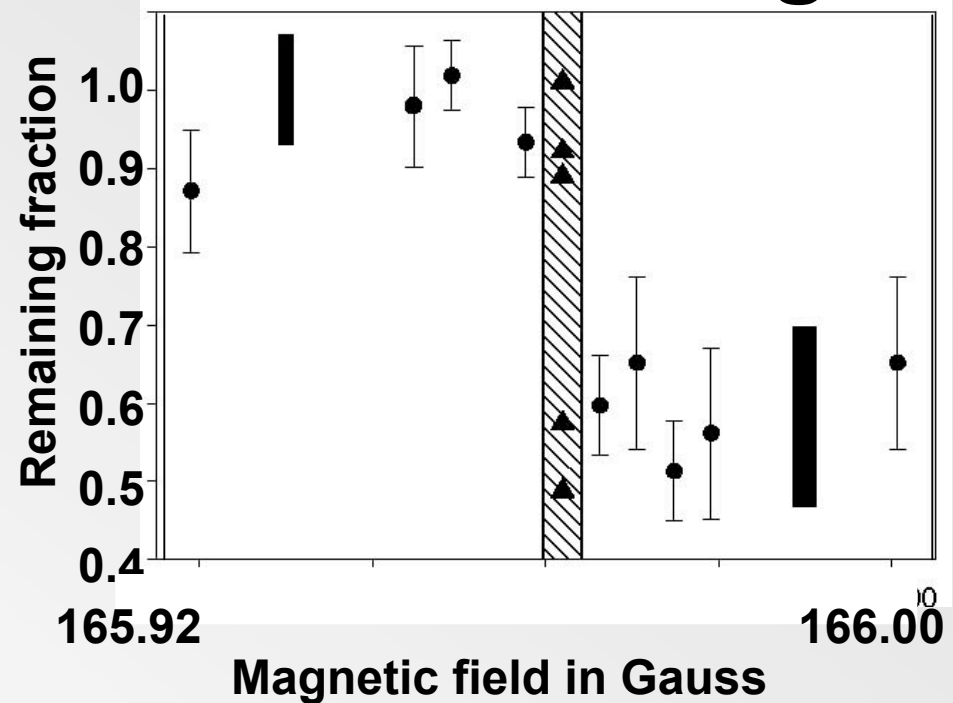
Dodd et al., PRA 54, 661 (1996)



## 2. Experiments with Bosonic $^{85}\text{Rb}$ : Probing Critical Mean-Field Strength

Roberts et al., PRL 86,  
4211 (2001)

- Mean-field prediction confirmed by experiment.
- Underlying physical mechanism? Three-body recombination.
- Physical picture? Connection between MF GP eq. and many-body Hamiltonian?



more negative  $a_s$

$$(N-1)a_s/a_{ho} \sim -0.58$$

## 2. Mean-Field Equation Derived from Many-Body Hartree Wave Function

- Many-body Hamiltonian for N bosons under confinement:

$$H = \sum_{j=1}^N \left[ \frac{-\hbar^2}{2m} \nabla_{\vec{r}_j}^2 + \frac{1}{2} m \omega^2 \vec{r}_j^2 \right] + \sum_{j < k}^N V_{aa}(\vec{r}_j - \vec{r}_k) \quad \text{SW, HS, ...}$$

- Hartree product (restricted Hilbert space):

$$\psi(\vec{r}_1, \dots, \vec{r}_N) = \prod_{i=1}^N \phi_a(\vec{r}_i)$$

- ZR atom-atom potential:

$$V_{aa}(\vec{r}) = \frac{4\pi \hbar^2 a_{aa}}{m} \delta^{(3)}(\vec{r}) \propto U_{aa} \delta^{(3)}(\vec{r})$$

- Gross-Pitaevskii (GP) equation for “single atom”:

$$\left( -\frac{1}{2} \nabla_{\vec{r}}^2 + \frac{1}{2} \vec{r}^2 + U_{aa} (N - 1) |\phi_a(\vec{r})|^2 \right) \phi_a(\vec{r}) = \epsilon_a \phi_a(\vec{r})$$

Single atom feels effective potential/mean-field created by the other N-1 atoms.

## 2. Mechanical Instability or Collapse of Trapped Bose Gas

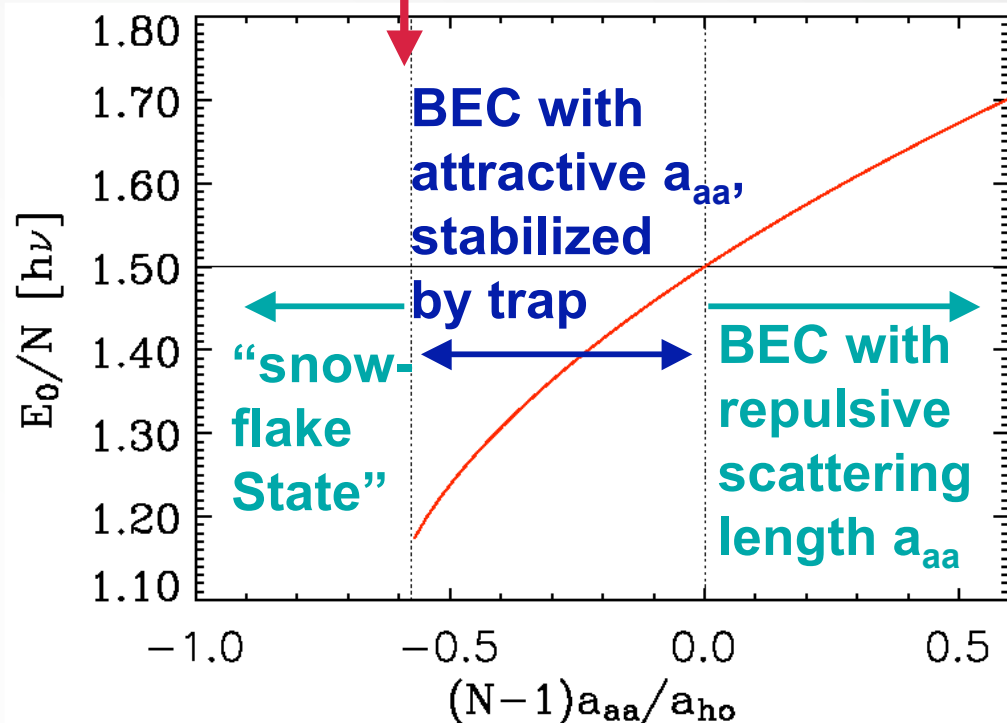
$$\left( -\frac{1}{2}\nabla_{\vec{r}}^2 + \frac{1}{2}\vec{r}^2 + \underbrace{U_{aa}(N-1)|\phi_a(\vec{r})|^2}_{1 \text{ parameter}} \right) \phi_a(\vec{r}) = \epsilon_a \phi_a(\vec{r})$$

1 parameter

Single particle orbital

No pos. E solution to GP eq.

Non-linearity



Why does ZR potential work?

At collapse point,  
 $n(0)|a_{aa}|^3 \ll 1$   
 (small parameter;  
 long wave length  
 approximation justified).

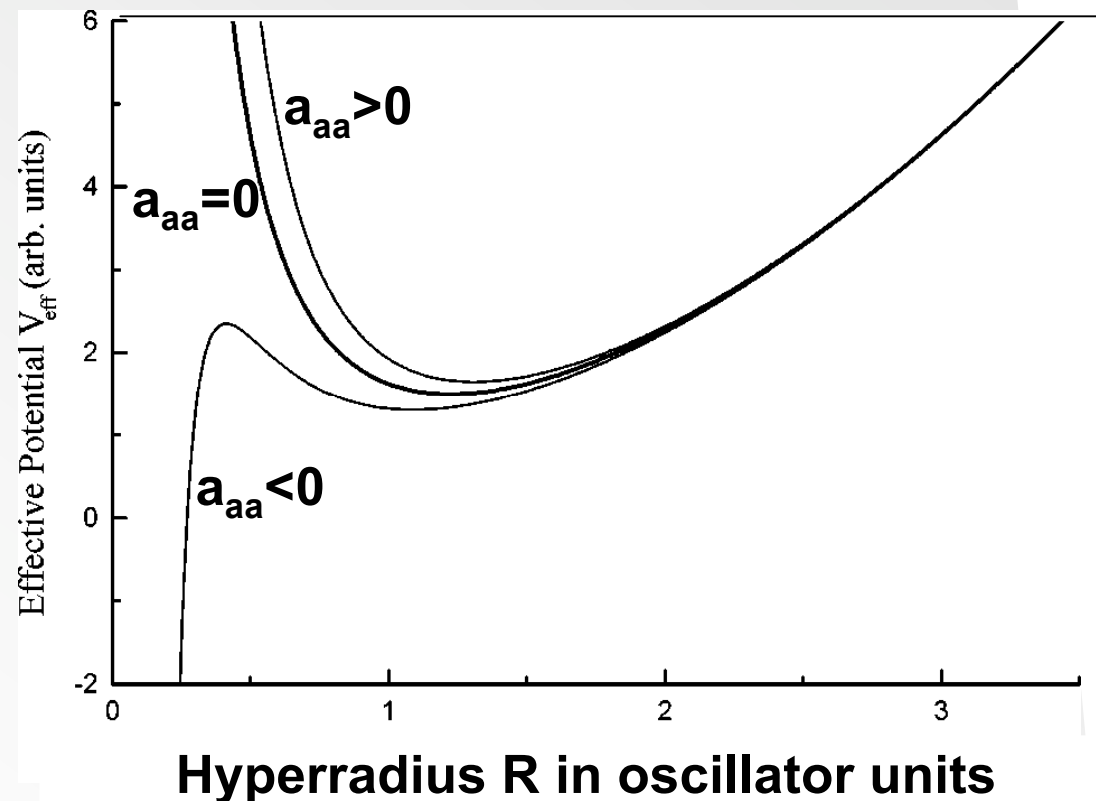


## 2. Interpretation within Hyperspherical Framework: Linear Schroedinger Eq.

- Coordinates: 1 hyperradius  $R$ ,  $(3N-4)$  hyperangles  $\Omega$ .
- Many-body symmetrized variational wave function:  $F(R)\Phi(\Omega)$ .
- Sum of two-body delta-function interactions.
- Effective potential:  $V_{\text{eff}}(R) = c_1/R^2 + c_2R^2 + c_3a_{aa}/R^3$

**Collapse prediction within ~20% of GP equation and experiment.**

**Bohn, Esry and Greene, PRA 58, 584 (1998)**



## 2. Hyperspherical Coordinates: Three Step Approach

$$M = Nm$$

$$\Psi_{NI}(\vec{r}_1, \dots, \vec{r}_N) = \underline{G(\vec{R}_{CM})} \underline{F(R)} \underline{\Phi(\vec{\Omega})}$$

$$R^2 = \frac{1}{N} \sum_{i=1}^N (\vec{r}_i - \vec{R}_{CM})^2$$

$$H_{NI} = \underline{H_{CM}} - \frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial R^2} + \frac{3N-4}{R} \frac{\partial}{\partial R} \right) + \frac{\Lambda^2}{2MR^2} + \frac{1}{2}M\omega^2 R^2$$

$$H_{CM} = \frac{-\hbar^2}{2M} \vec{\nabla}_{CM}^2 + \frac{1}{2}M\omega^2 R_{CM}^2$$

$$E_{CM} = (2n_{CM} + l_{CM} + 3/2)\hbar\omega$$

1: Remove CM degrees of freedom.

2: Solve hyperangular equation (gives  $V_{\text{eff}}(R)$ ).

$$\Lambda^2 \Phi_{\lambda,\mu}(\vec{\Omega}) = \hbar^2 \lambda(\lambda + 3N-5) \Phi_{\lambda,\mu}(\vec{\Omega})$$

3: Solve hyperradial equation.

With interactions:  
delta-function  
interactions can be  
treated analytically for  
lowest hyperspherical  
harmonic.

# 3. Similarly: Analyze Stability of Multi-Component Fermi Gases

- Simple variational wave function (ideal gas nodal surface) applied to multi-component Fermi gas with “bare” zero-range interactions (all s-wave scattering lengths equal) predicts collapse at:

- 2-component gas:  $(k_F a_{aa})^3 \sim -1.81$  (not small compared to 1)

- 3-component gas:  $(k_F a_{aa})^3 \sim -0.23$

- 4-component gas:  $(k_F a_{aa})^3 \sim -0.067$  (small compared to 1)

- However: Two-component Fermi gas is found to be stable (experiment plus Monte Carlo).



How many components are needed to get instability? Proper theoretical framework?

- Multi-component Fermi gas with one atom per component (Bose gas): Unstable at unitarity.

# 3. Two-Component Fermi Gas: Renormalization of Interaction

- Many-body anti-symmetrized wave function:  $F(R)\Phi(\Omega)$ .
- Effective potential:  $V_{\text{eff}}(R) = c_1/R^2 + c_2R^2 + c_3k_F a_{aa}/R^3$

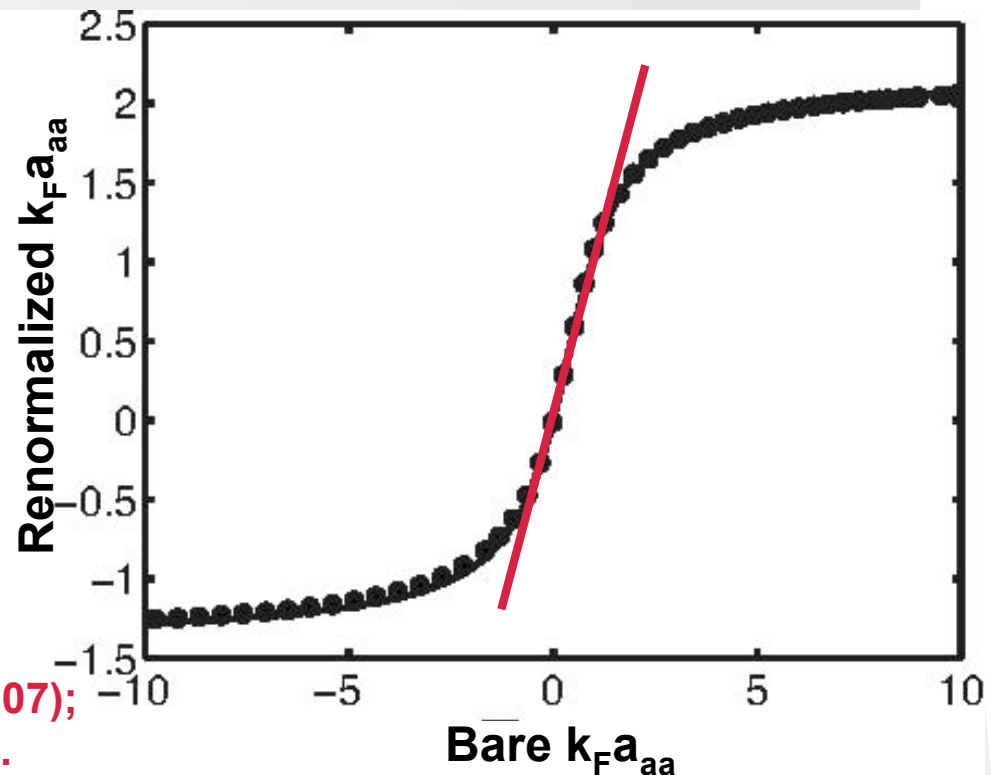
At collapse point,  $k_F|a_{aa}|\sim 1$ .  
ZR potential no longer applicable.

Renormalize  $k_F|a_{aa}|$  so as to reproduce two-particle spectrum.

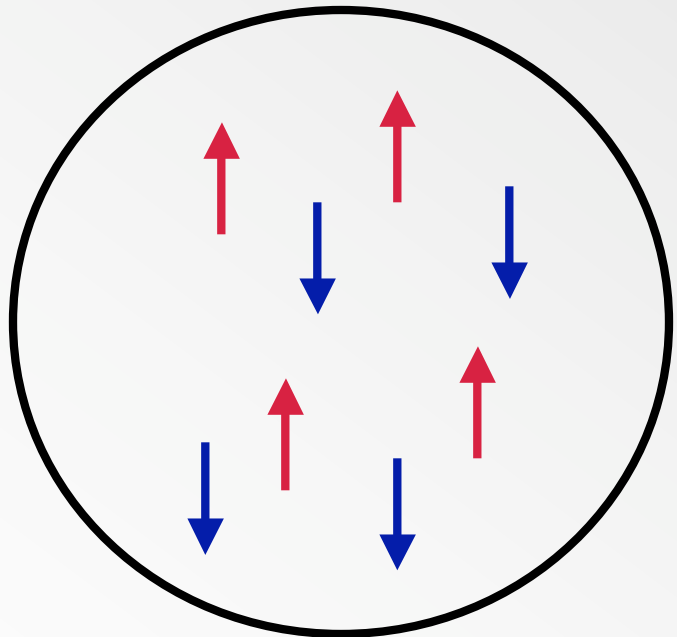
Use renormalized  $k_F|a_{aa}|$  in hyperspherical framework.

Rittenhouse, Cavagnero, von Stecher and Greene, PRA 74, 053624 (2006);  
von Stecher and Greene, PRA 75, 022716 (2007);  
Rittenhouse and Greene, physics/0702161v2.

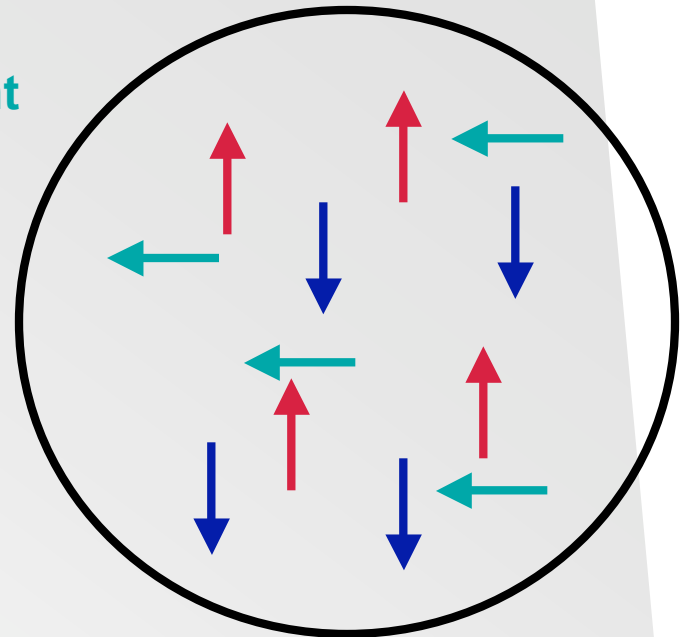
Renormalized ZR potential:  
For  $|a_{aa}|\rightarrow\infty$ :  $\text{const}/R^2$



# 3. Why Collapse for More Components? “Counting Argument”



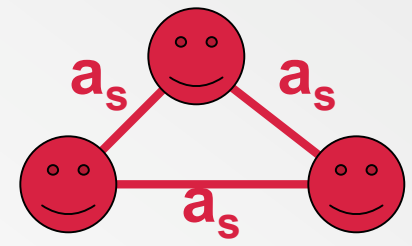
Add a third component  
(equal spins:  
non-interacting;  
unequal spins:  
attractive interaction)



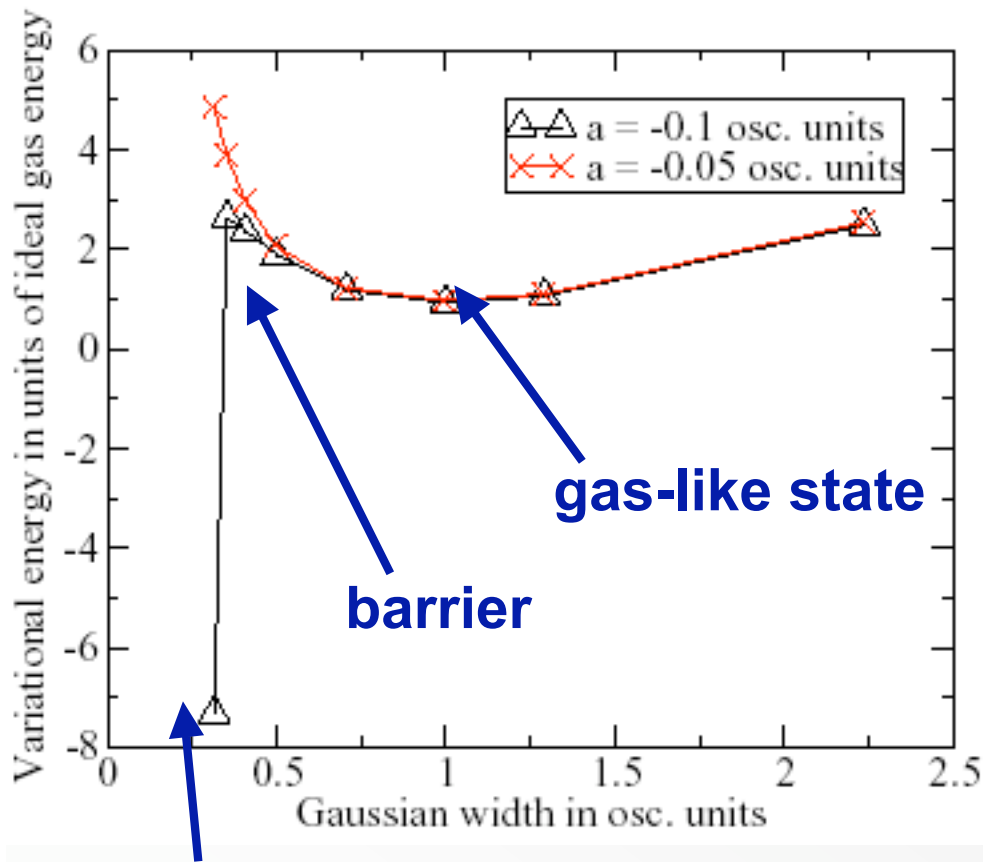
Fermi statistics (repulsion):  
12 equal spin pairs.  
**Attraction:**  
16 unequal spin pairs.



Fermi statistics (repulsion):  
18 equal spin pairs.  
**Attraction:**  
48 unequal spin pairs.

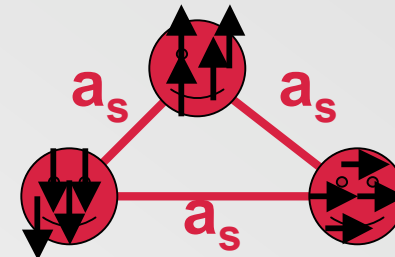


# 3. Many-Body Variational MC Treatment of Three-Component Fermi System



cluster state (similar to Bose gas with negative scattering length)

- Four fermions per component:



- Square well interaction potential with range  $r_0 = 0.01 a_{ho}$ .
- Variational wave function with one parameter  $b$  that determines size:  $\exp(-0.5(r/b)^2)$
- Negative energy state exists if  $|a_{aa}|$  "large".
- Peak of barrier at length scale a few times  $|a_{aa}|$ ,  $|a_{aa}| \ll r_0$ .

# Summary and Outlook

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- **Broad introduction to cold atom physics: Many experiments and strong interactions among people from different communities.**
- **Revival of few-body physics: Largely due to Efimov physics (fourth lecture).**
- **Connections between microscopic and macroscopic worlds: Stability of Bose and Fermi gases within many-body framework.**
- **Next lecture:**
  - **Few-body techniques and calculations.**



# General Summary of Field of Cold Atom Physics

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- Interaction strengths can be controlled (Fano-Feshbach resonance).
- Confinement can be designed (lattice, quasi-1d,...).
- **Fundamental physics question:**
  - Strongly-interacting system.
  - Multi-component systems.
  - Equal- and unequal-mass systems.
  - Efimov physics.
  - Dipolar systems.
- **Applications:**
  - High precision measurements of fundamental constants.
  - Navigational devices.
  - Quantum computation and quantum simulation.