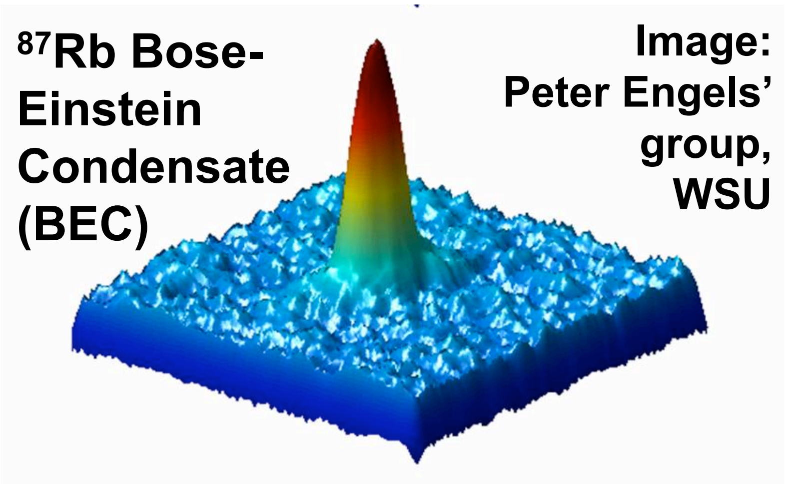
Bosonic Few-Body Systems: What is Different?

(compared to fermions…)



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Outline of This Talk

- Efimov scenario: Three-body system.
  - Efimov's proposal (~1970/71): infinitely many three-body bound states.
  - First experimental signatures in atomic gases (2006).

- Extended Efimov scenario: Four-body system.
  - Not infinitely many four-body bound states!
  - Instead: Fourth atom "tagged on" to Efimov trimer.

- Beyond four?

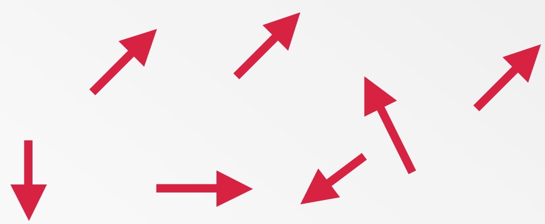
- Hardcore Bose gas.

Throughout: Identical bosons interacting through short-range two-body potential (s-wave interactions).

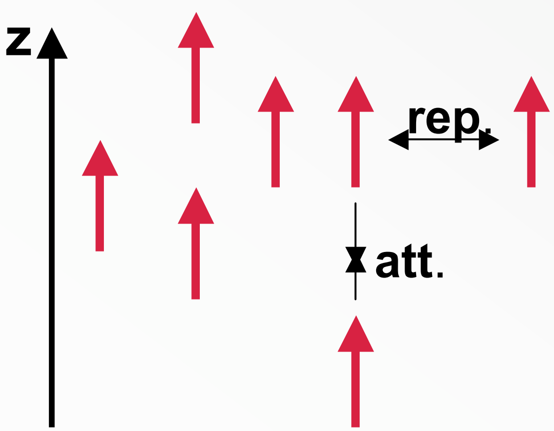
Cold versus Ultracold?
Isotropic versus Anisotropic?

WEDNESDAY:

Ensemble of magnetic or electric dipoles:

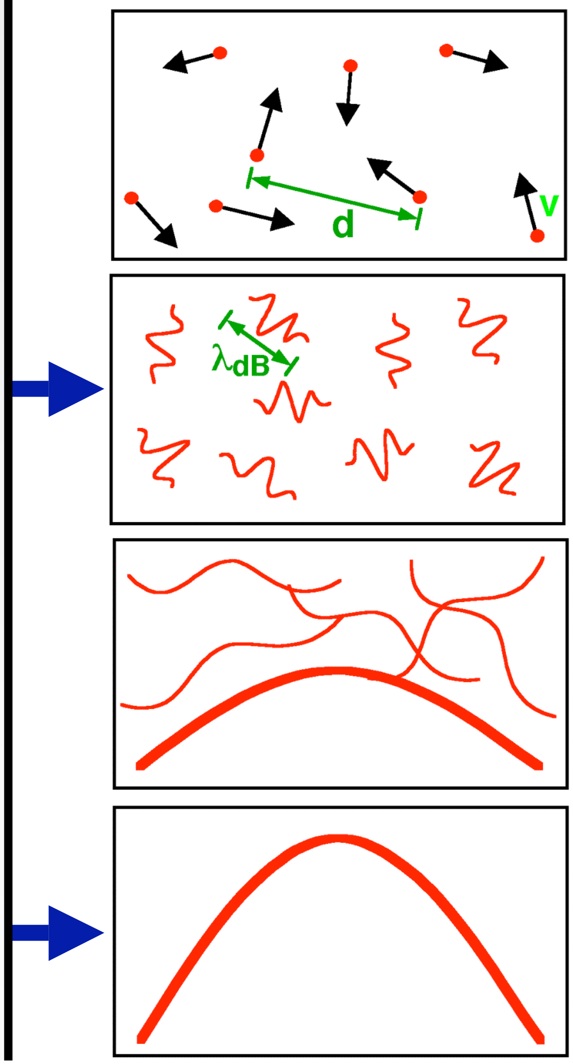


Dipoles can be aligned through application of external B- or E-field:



TODAY:

Image credit:
Ketterle group at MIT



High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

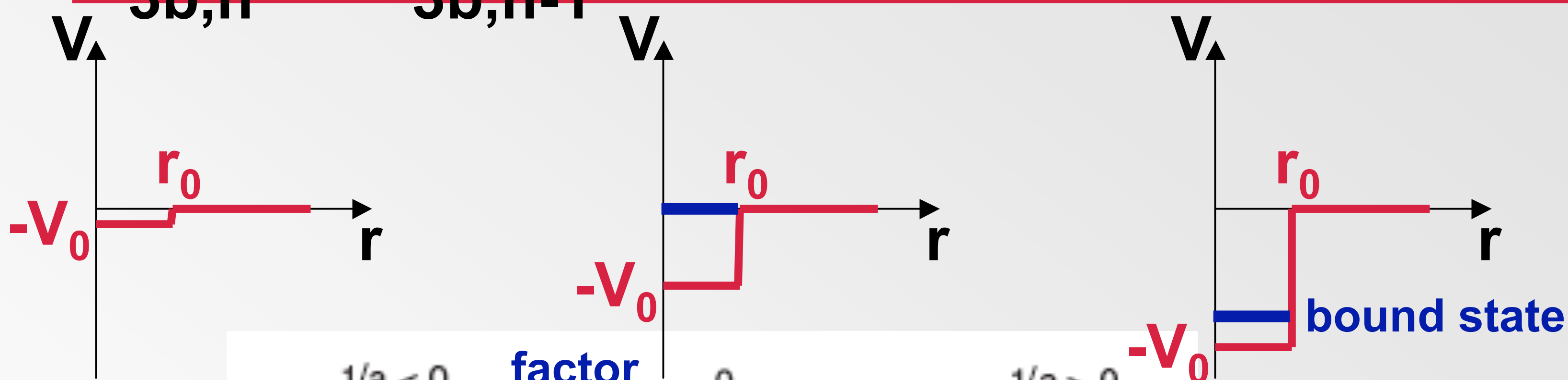
Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

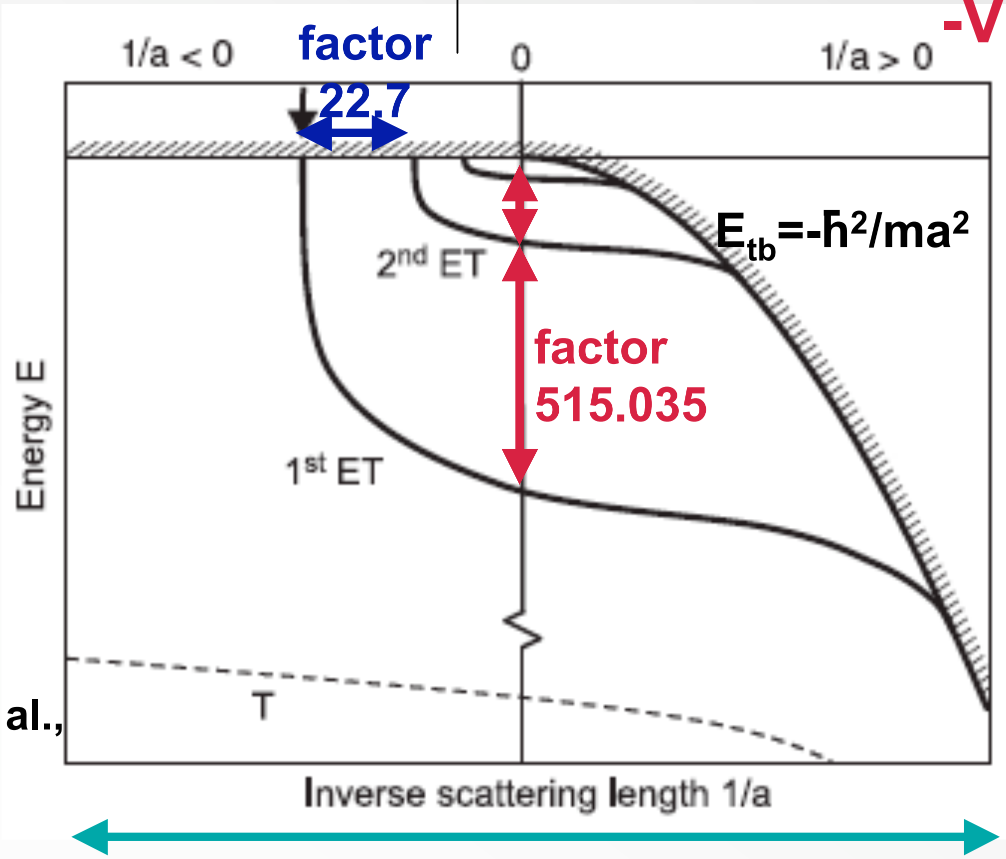
$T=T_{crit}$: Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0: Pure Bose condensate
"Giant matter wave"

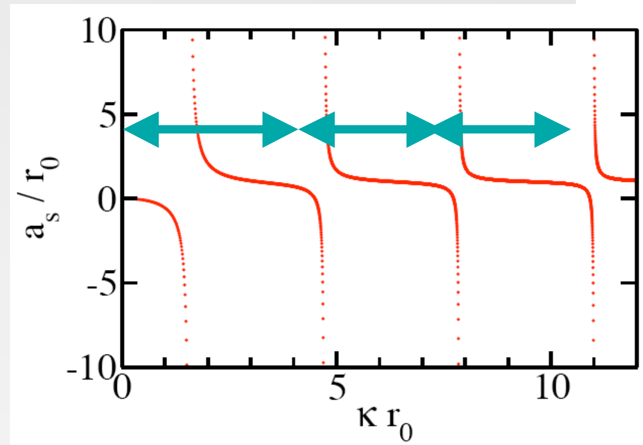
Efimov Scenario:
$E_{3b,n} = E_{3b,n-1} / 515.035$

V. Efimov, Phys. Lett. B 33, 563 (1970); Sov. J. Nucl. Phys. 12, 589 (1971).





Kraemer et al., Nature 440, 315 (2006).



$\kappa \propto \sqrt{V_0}$

Geometric convergence of three-body states toward zero for $1/a_s=0$.

Efimov Scenario:
More Background and Implications

- **Prediction for three identical bosons (no trap):**
  - If two-body system supports no bound state ($a_s \to \infty$), three-body system supports infinitely many.

- **At unitarity:**
  - Hyperradial potential curve $V(R) = (s_0^2 - 1/4) / (2MR^2) = (-1.0062378^2 - 1/4) / (2MR^2)$.
  - Actual values of three-body energy and size determined by underlying two-body interaction.
  - Nevertheless, discrete scale invariance: $\langle R \rangle_n = \langle R \rangle_{n-1} \exp(\pi/s_{0,R}) = 22.7 \langle R \rangle_{n-1}$ and $E_n = E_{n-1} \exp(-2\pi/s_{0,R}) = E_{n-1} / 515$.

- **For finite a_s:**
  - $V(R)$ deviates from $1/R^2$ behavior for $R > a_s$ and number of Efimov states is roughly $N_b \sim s_{0,R}/\pi \ln(|a_s|/r_0)$.

Why Was $^4He_3$ Considered the Most Promising Candidate?

- ^{4}He dimer:
  - $a_s \sim 100$Å, $r_{eff} \sim 10$Å.
  - Single bound state: $E_{2b} \sim 1$mK (note: $E/N_{bulk} \sim 10$K!!!)
  - Number of Efimov states ~ 0.7 to 0.8.
- ^{4}He trimer has two L=0 and no L>0 bound states:
  - Ground state ("deeply bound"): $E_{3b} \sim 100$mK.
  - Excited state has Efimov character (bound state goes away when depth of two-body potential is increased artificially): $E_{3b^*} \sim 1$mK.
- But: Even if $^4He_3^*$ were detected unambiguously, how to probe/proof its Efimov character?

Schoellkopf and Toennies, JCP 104, 1155 (1996); Nielsen et al., JPB 31, 4085 (1998); Blume and Greene, JCP 112, 8053 (2000); Lee et al., JPB 34, L203 (2001);…

Observing Universal Regime of Three-Body Interactions Experimentally?

Measurement based on $dn/dt = -L_2 n^2 - L_3 n^3 - L_4 n^4$
(n density; L_N averaged inelastic loss rate associated with collisions of N bodies; typically L_2 small and $L_3 > nL_4$):

- Scattering length needs to be changed by factor of $(22.7)^2 = 515$ to see two features with predicted spacing [ideally want three: $(22.7)^3 = 1.2\text{x}10^4$].

- Temperature must be extremely low: $k_B T < \hbar^2/ma_s^2$ (high T washes out effects: thermal averaging).

- Since L_3 shows overall scaling of $|a_s|^4$, variation of a_s by factor of 515 implies change of L_3 by factor of $2.7\text{x}10^6$.

  But need to measure small change compared to overall scaling (need dynamic range of at least 10^8).

See Greene, Physics Today, March 2010, pp. 40 for very nice and accessible discussion.

Implications of Three-Body Loss Coefficient L_3

- **Assuming absence of any other length scale, dimensional analysis gives overall scaling of L_3 with $|a_s|^4$:**
  - **Since $dn/dt \propto L_3 n^3$, L_3 has units of length^6/time.**
    Time $\propto$ h/energy.
    Energy $\propto h^2/m|a_s|^2$.
    Plugging in: $L_3 = C(a_s)h|a_s|^4/m$, where $C(a_s)$ is dimensionless logarithmically periodic function that contains signatures of Efimov states.

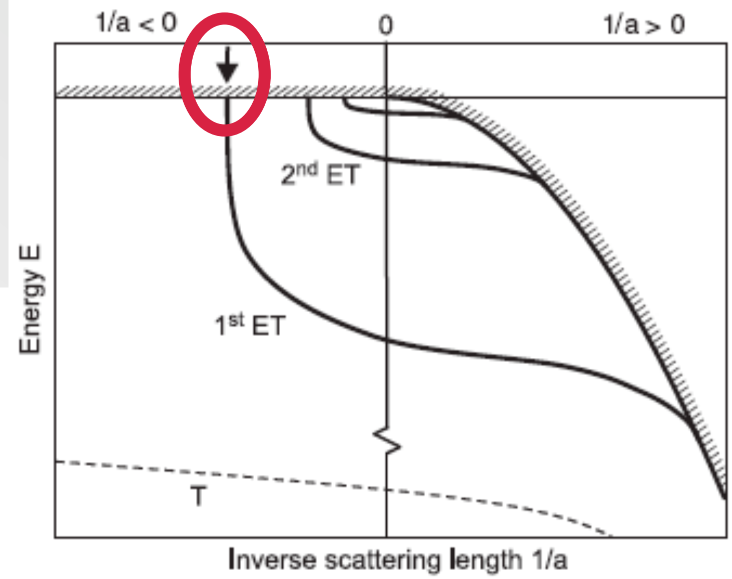
- **Large $|a_s|$ implies large losses (instability of BEC for large $|a_s|$; a_s positive or negative).**
- **But observation of Efimov physics requires large $|a_s|$, in particular also for negative a_s.**
- **To probe Efimov physics, most experimental groups work with non-condensed samples.**

Nice review: Braaten and Hammer, Phys. Rep. 428, 259 (2006).

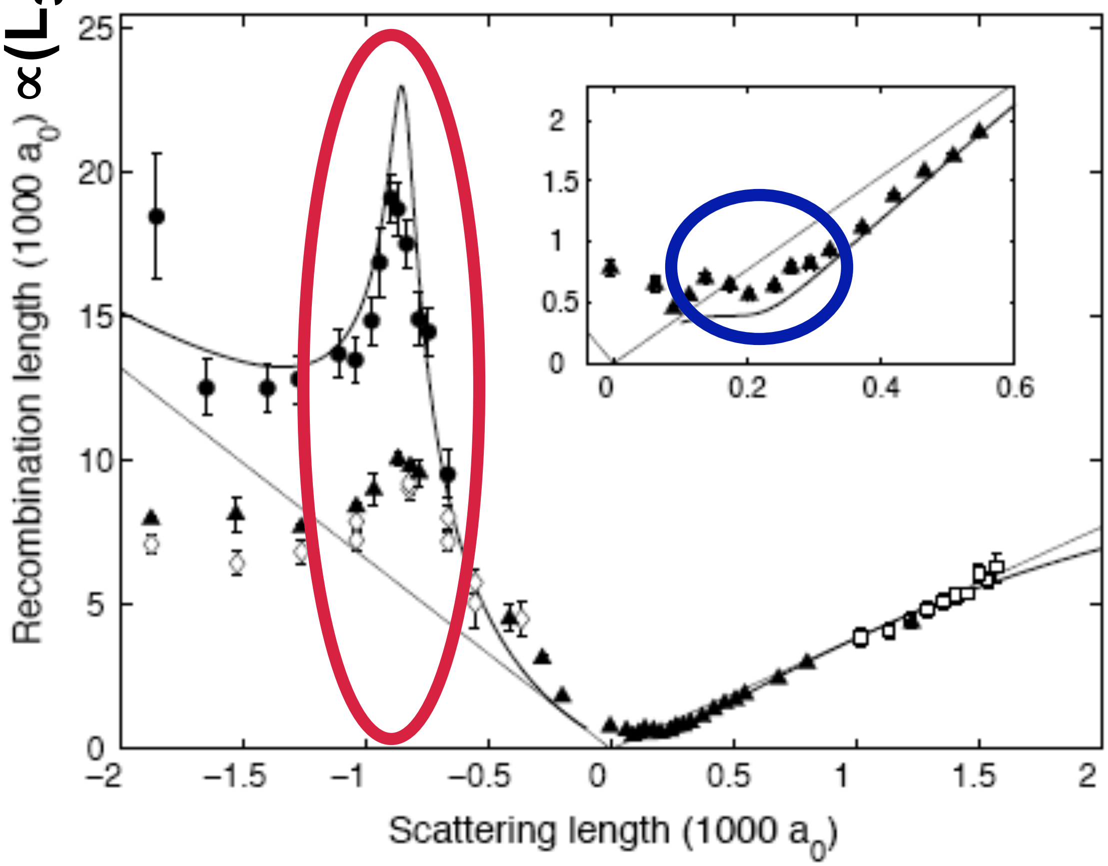
Experimental Evidence of Efimov Scenario in Cold Atomic Gases

Special value:

a_i^{3b} (<0): i^{th} Efimov trimer state hits zero and becomes unbound (3=1+1+1).



Kraemer et al., Nature 440, 315 (2006).

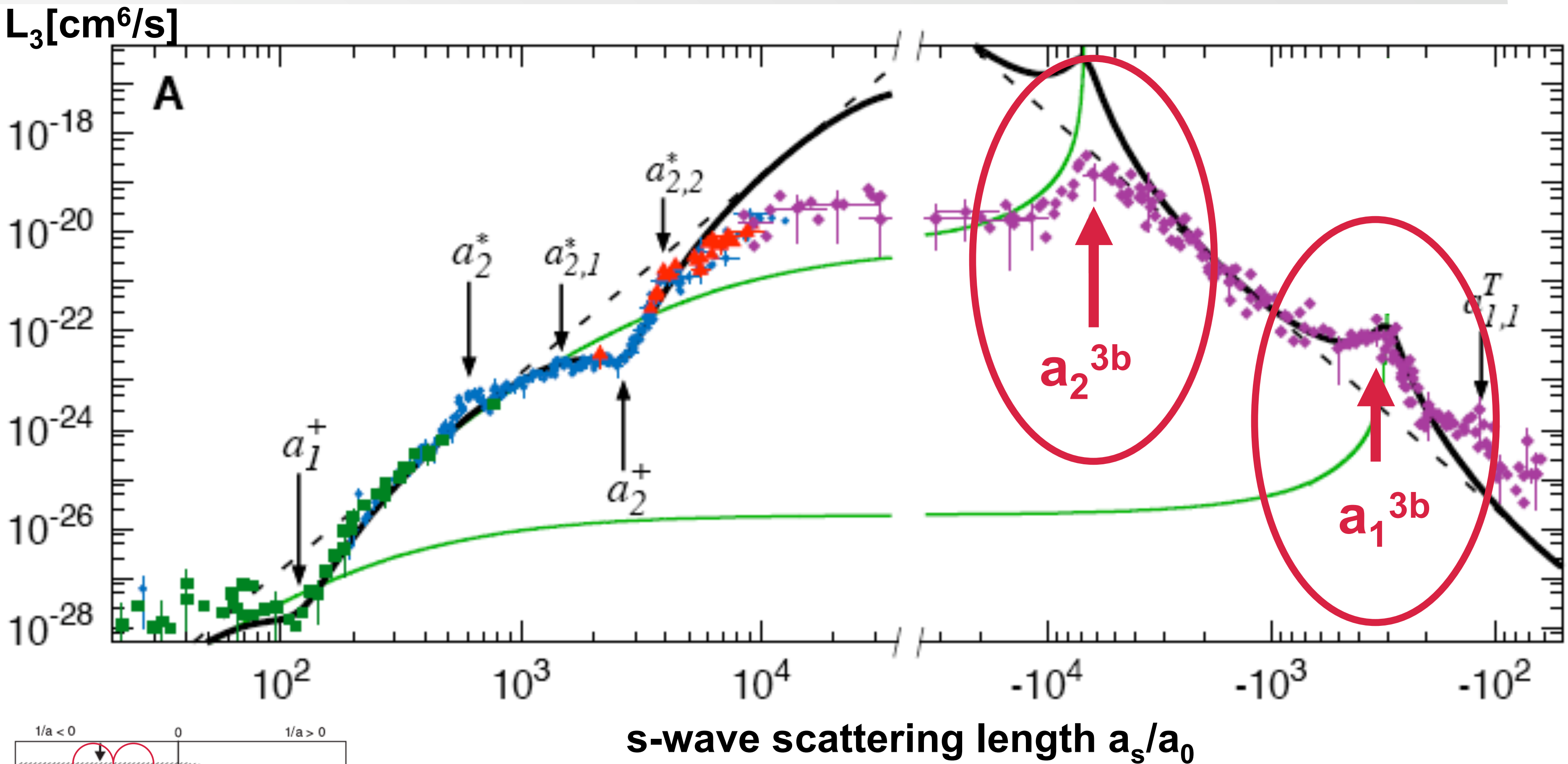


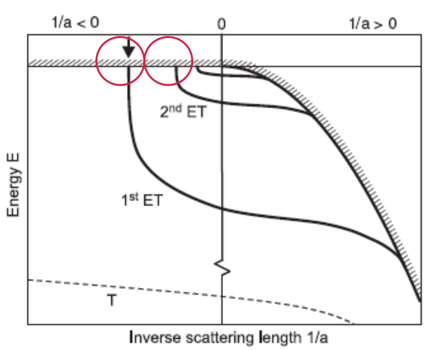
Filled circles: T=10nK.
Filled triangles and open diamonds: T=200nK.

Special value:

$a^{3b,min}$ (>0): minimum in three-body recombination rate (constructive inter-ference of two pathways).

Experimental Confirmation of Efimov Scenario in Cold Atomic Gases





exp. a_2^{3b}/a_1^{3b}=22.2 (th. 22.7)

Pollack et al., Science 326, 1683 (2009).

Going Beyond Three: What are the Questions?

- Ratio between three-body energies is universal (~515):
  - Absolute position of three-body energy determined by three-body parameter (typically taken as a fitting parameter when analyzing experimental data).

- Efimov effect for tetramer? No (no sequence of infinitely many four-body bound states).

- Do four-body bound states exist?
- If yes, is a four-body parameter needed to nail their position or are the four-body states somehow universally tight to Efimov trimers?

- What about 5-body, 6-body,…?

Extended Efimov Scenario for Three- and Four-Boson Systems

Theoretical predictions:

Two four-body states for each Efimov trimer
[Platter et al., PRA 70, 052101 (2004)].

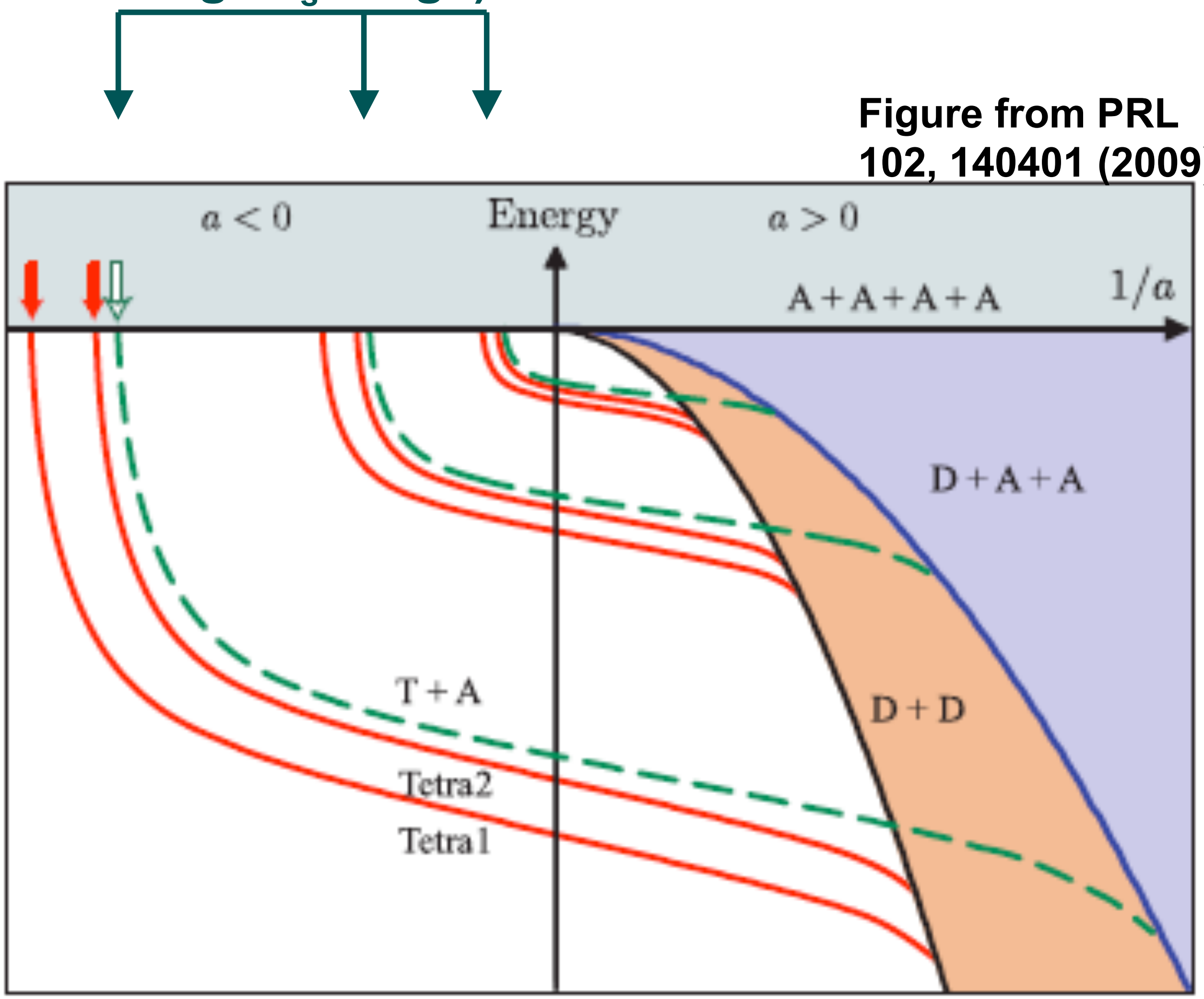
$a_{Tetra1} \approx 0.49 a_{Trimer}$
[Hanna and Blume, PRA 74, 063604 (2006)]

$a_{Tetra1} \approx 0.43 a_{Trimer}$
$a_{Tetra2} \approx 0.9 a_{Trimer}$
[von Stecher, D'Incao, Greene, Nature Physics 5, 417 (2009)]

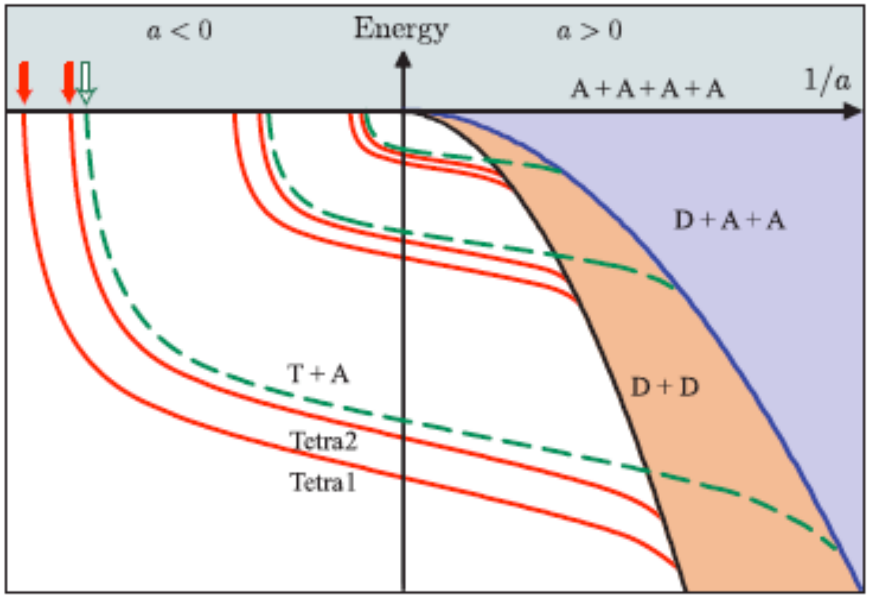
More special values…

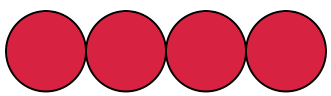
"usually" (requires large a_s range)

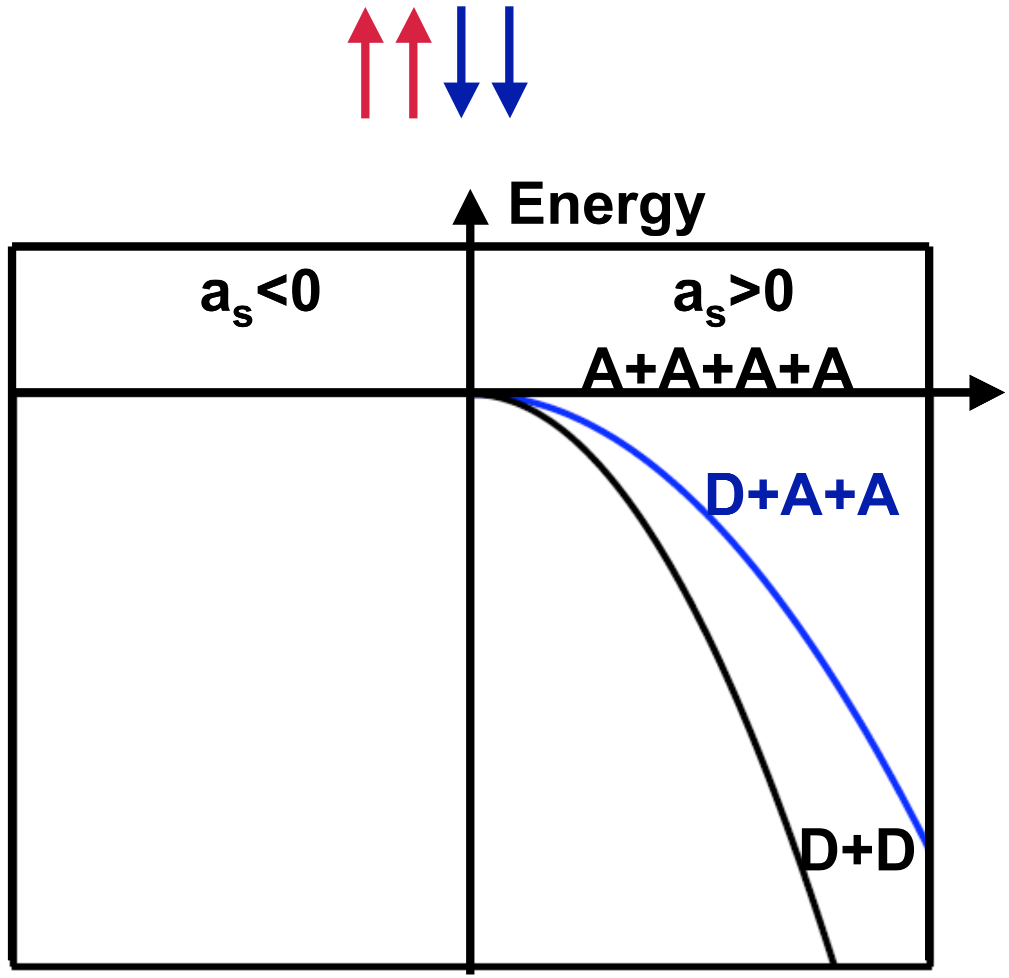
Figure from PRL 102, 140401 (2009)



Four-Boson versus Four-Fermion System



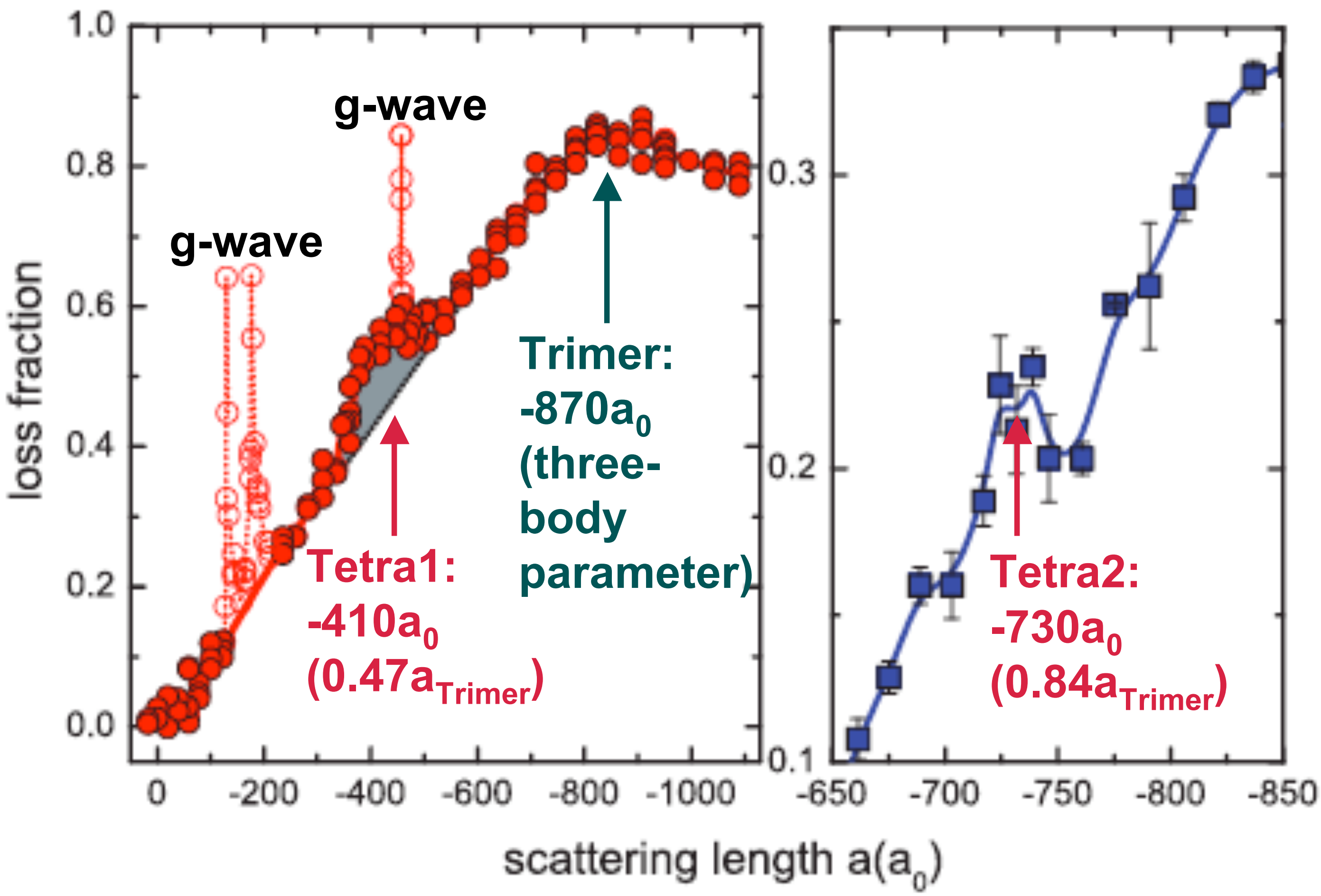


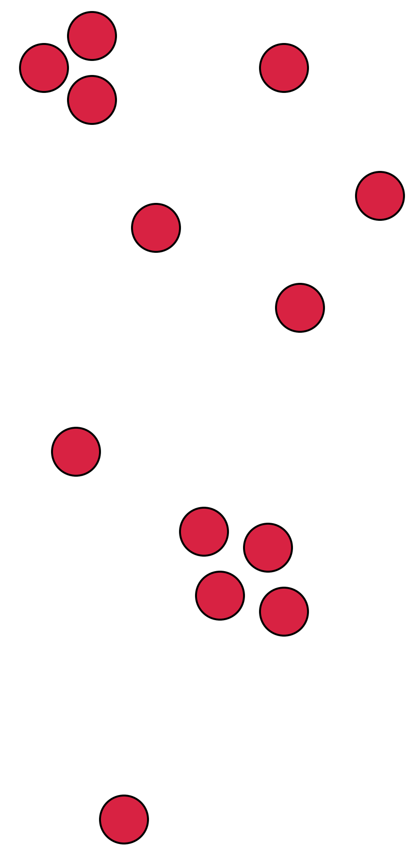


Measurement of Loss Rate for Non-Degenerate Bosonic ^{133}Cs Sample

First measurement of universal 4-body physics (probe of Efimov physics)

Ferlaino et al., PRL 102, 140401 (2009).





How to stabilize these delicate trimers and tetramers?

How To Determine the a_s's at Which N=3 and 4 Systems Become Unbound?

- Effective field theory (Platter and Hammer).

- Hyperspherical coordinate approach (von Stecher, D'Incao and Greene).

- Our approach:
  - Consider pairwise van der Waals interaction (no trap) and estimate at which point system becomes unbound.
  - Calculate E(m) for different N (lighter mass m implies weaker binding; m can be thought of as coupling strength g, $g=4m\varepsilon\sigma^2/h^2$ for Lennard Jones potential).
  - E(m) determined by DMC method (straightforward, except need to be extremely careful close to threshold).

Diffusion Quantum Monte Carlo (DMC): Zero-Temperature Method

Goal: Determine ground state properties of many-body system at T = 0, that is, solve:

$$H\phi_0(\vec{R}) = E_0\phi_0(\vec{R})$$

Introduce imaginary time:

$$\tau = \frac{i}{\hbar}t$$

Time-dependent SE:

$$-\frac{\partial\Psi(\vec{R},\tau)}{\partial\tau} = (T + V - E_T)\Psi(\tau,\vec{R})$$

Propagate to large τ:

$$\Psi(\vec{R},\tau) \rightarrow c_0\phi_0(\vec{R}) + \sum_{k=1}^{\infty} c_k\phi_k(\vec{R})\exp[-(E_k - E_T)\tau]$$

Small time step Δτ:
τ = n Δτ, where n large.

Algorithm filters out nodeless ground state.

Diffusion Quantum Monte Carlo With Guiding Function

See, e.g., Reynolds et al., JCP 77, 5593 (1982)

Use trial/guiding function ψ_T:

$$\frac{\partial(\Psi\psi_T)}{\partial\tau} = \underbrace{\frac{\hbar^2}{2m}\nabla^2(\Psi\psi_T)}_{\text{diffusion}} - \underbrace{\frac{\hbar^2}{m}\nabla\cdot(\Psi\psi_T\vec{F})}_{\text{drift}} - \underbrace{(E_L - E_T)(\Psi\psi_T)}_{\text{source/sink}}$$

Force:

$$\vec{F} = 2\psi_T^{-1}\nabla\psi_T$$

Local energy:

$$E_L = \psi_T^{-1}H\psi_T$$

Choose ψ_T so that variance of local energy is minimized:

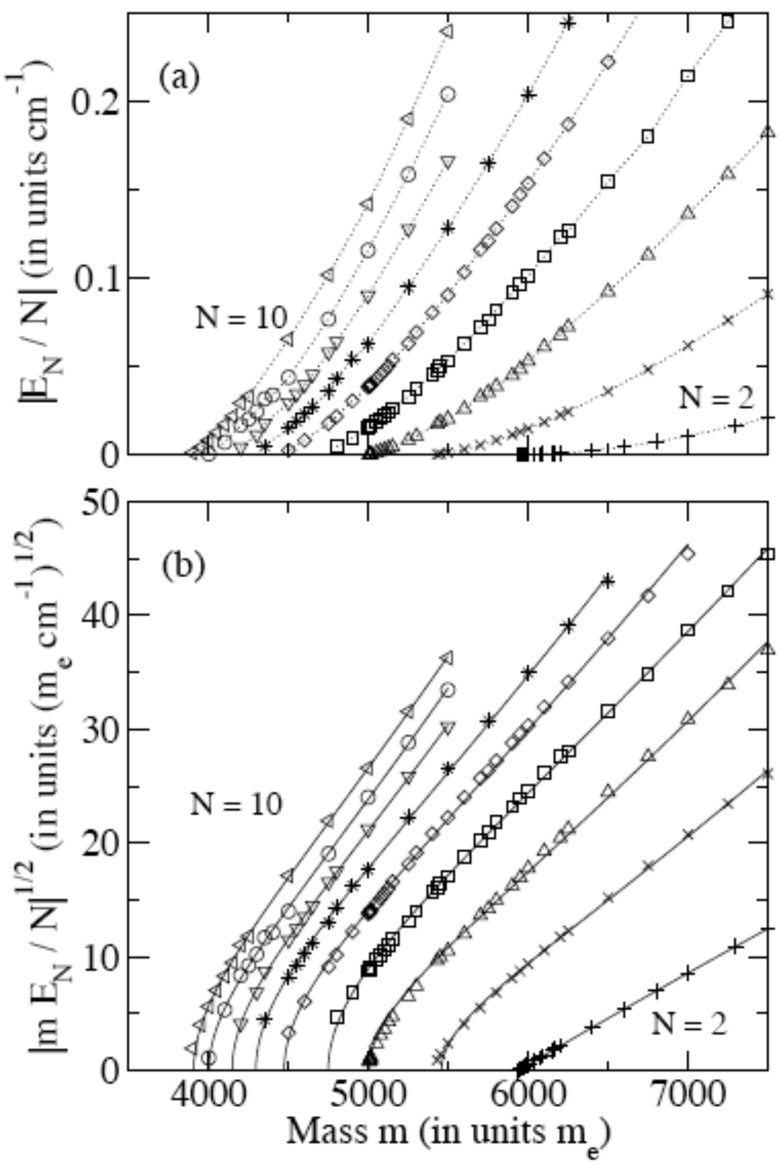
N<11: $\psi_T = \Pi_{i<j}\,\phi(r_{ij})$ where $\phi(r) = \exp[-\,p_{-5}r^{-5} - p_{-2}r^{-2} - p_0\ln(r) - p_1 r]$

N>10: $\psi_T = [\Pi_{i<j}\,\phi(r_{ij})]\,[\Pi_i\,\Theta(r_{i,cm})]$ where $\Theta(r) = [1+\exp((r-p)/w)]^{-1}$

Result:

Rigorous upper bound for E (if no systematic bias present).

DMC Results for Ground State Energy



Symbols: DMC energies.
Lines: Fit to DMC energy.

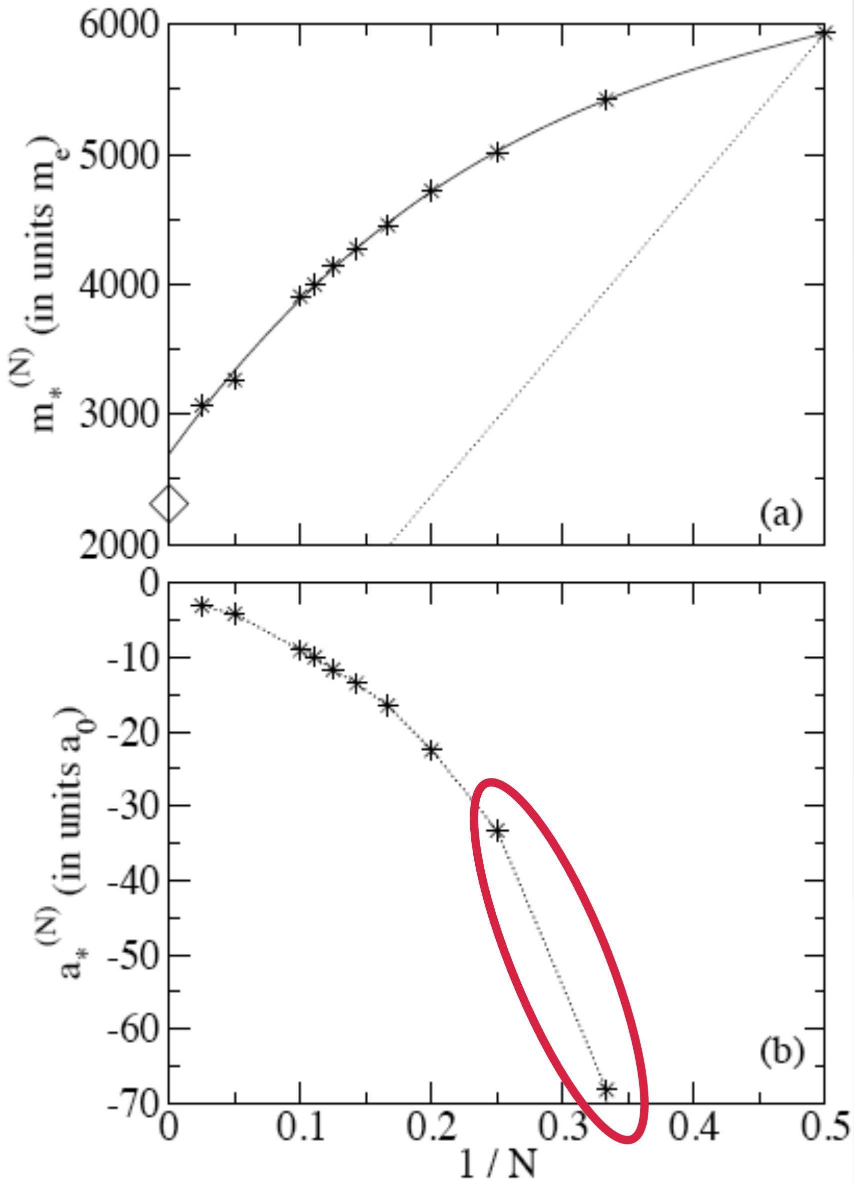
For N=2:
$$(m|E_2|)^{1/2} = \Sigma_{i=1,2}\, c_i (m-m_*)^i$$

For N>2:
$$(m|E_N/N|)^{1/2} = (m-m_*)^{1/2}\, \Sigma_{i=0,1,2}\, c_i (m-m_*)^i$$

m_*: mass at which system becomes unbound.

Hanna and Blume, PRA 74, 063604 (2006); see also von Stecher, to appear in JPB, for follow-up study.

Critical Coupling Strengths and Scattering Lengths



Pluses: Critical masses m_* determined by fit to DMC energies.

To convert critical masses into critical scattering lengths, perform two-body scattering calculation.

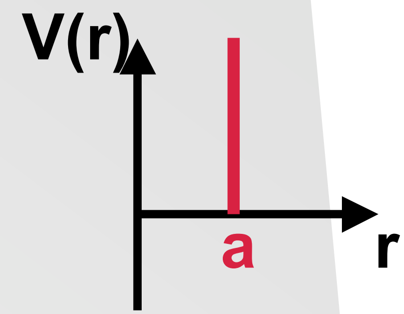
Ratio of $a_*(N=3)/a_*(N=4) \approx 0.49$.

Compares favorably with 0.43 value by von Stecher et al.

We find evidence that no 4-, 5-,… body parameters are needed.

Aside: When we performed these calculations, we did not aim at unravelling Efimov physics…

Condensed Bose Gas:
Does Three-Body Physics Play a Role?



Consider homogeneous system with density n interacting through hardspheres of radius a:

Energy per particle (in units of $\hbar^2/2ma^2$):

$$E/N = 4\pi\, na^3\, [1 + c_1\, (na^3)^{1/2} + c_2\, na^3 \ln(na^3) + c_3\, na^3 + \ldots]$$

Mean-field term (Bogoliubov, 1947)

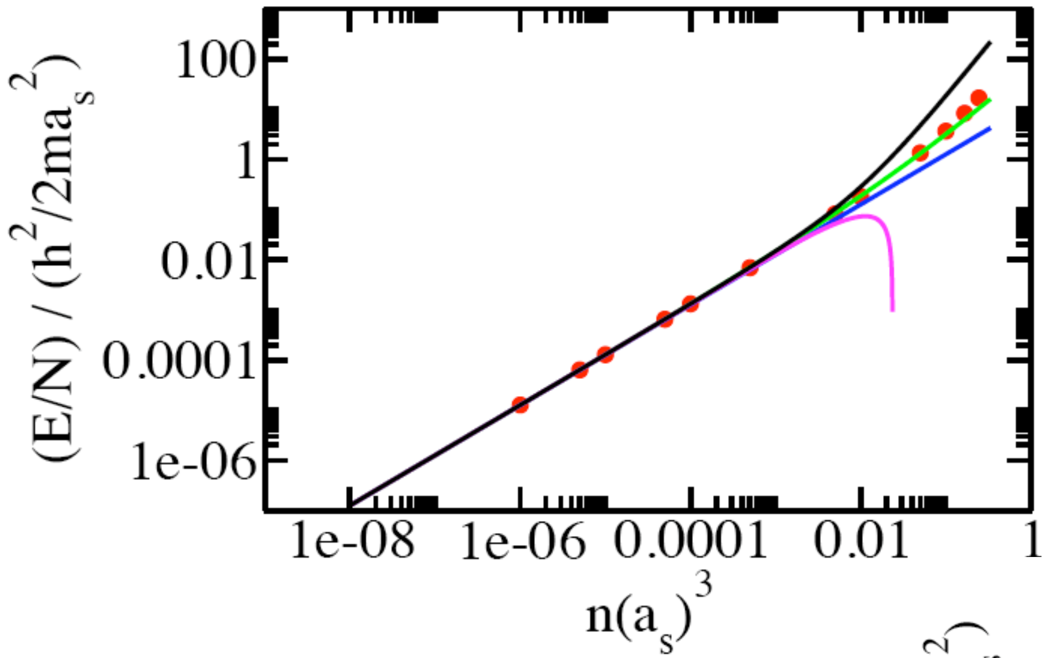
LHY (Lee-Huang-Yang) correction, 1957.
$c_1 = 128/15/\pi^{1/2}$

Wu correction, 1959.
$c_2 = 8(4\pi/3-3^{1/2})$

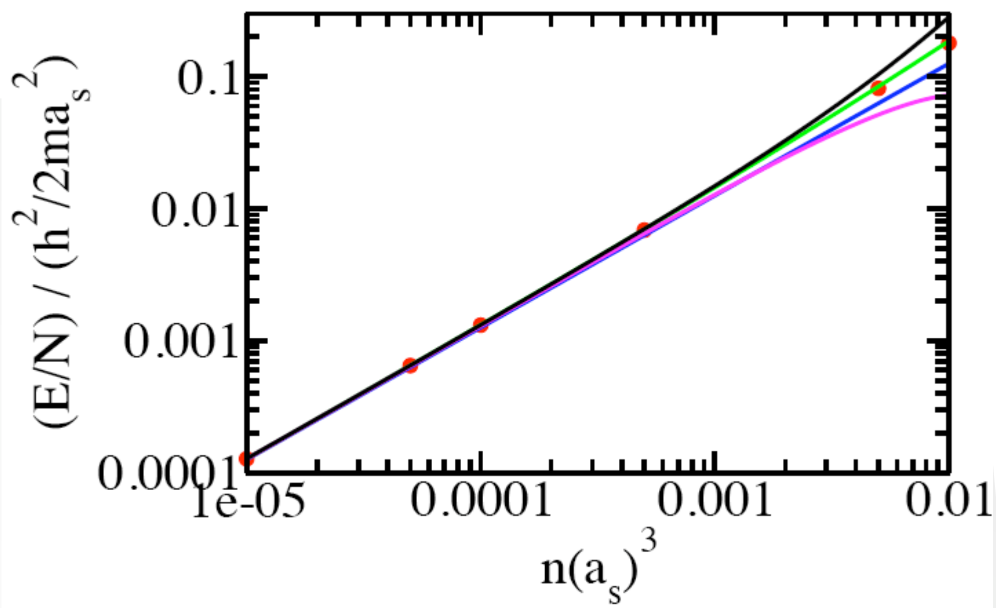
Three-body term discussed by Wu in 1957. 2008: c_3=167.31969(6)

[Tan, PRA 78, 013636 (2008); see also Braaten et al., Eur. Phys. J. B 11, 143 (1999)].

Comparison of na_s^3 Expansion with DMC Data for Hard Spheres



Red symbols: DMC data [Giorgini et al., PRA 60, 5129 (1999)].
Blue line: Mean-field (Bogoliubov).
Green line: MF + Lee-Huang-Yang.
Magenta line: MF + LHY + Wu.
Black line: MF + LHY + Wu + 3-body.



Observations:

1. MF + LHY correction works well.
2. Expansion not particularly nicely behaved.
3. Measurement of three-body parameter extremely hard.

Equivalent for Trapped System: GP Equation and Modified GP Equation

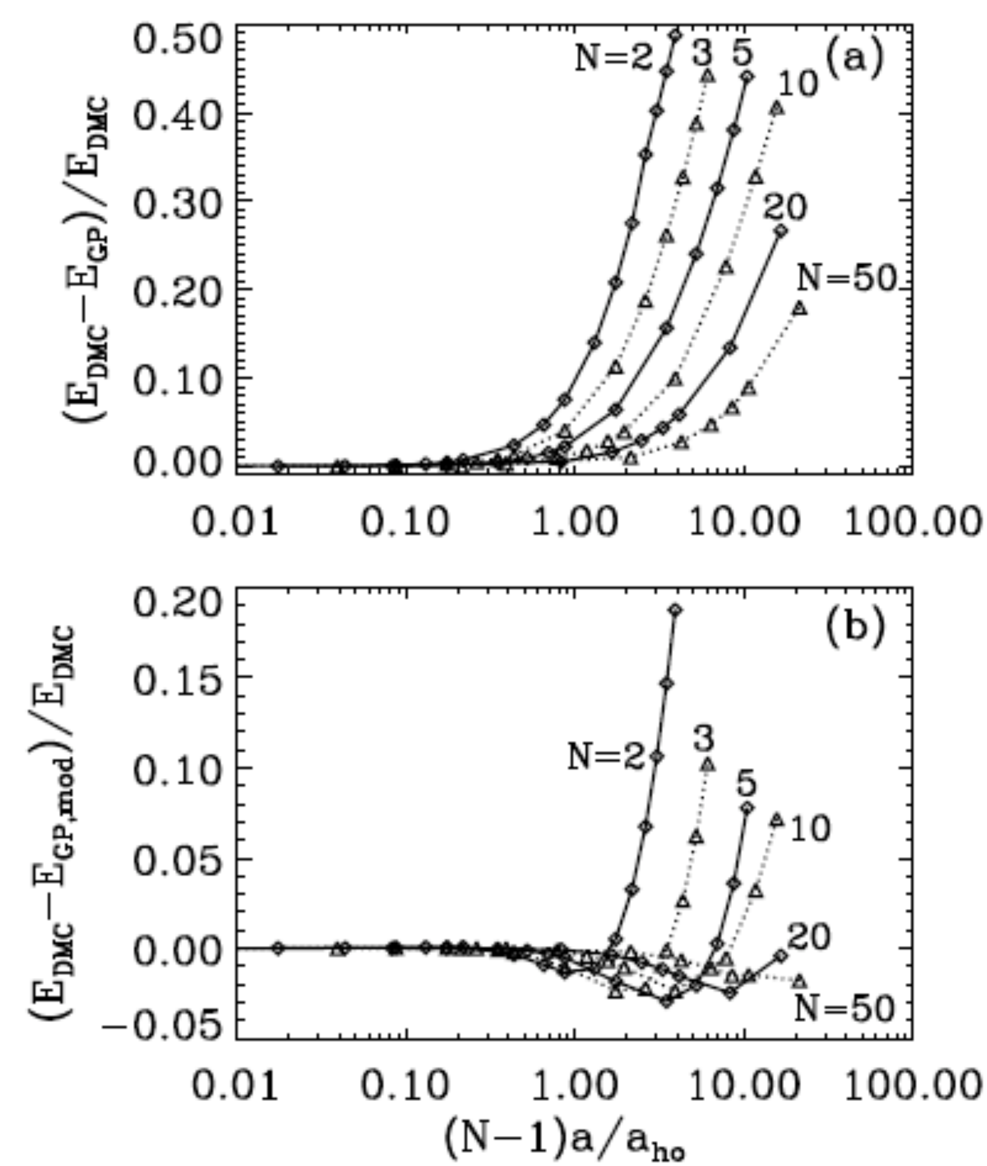
- **Mean-field GP equation:**

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_{ho}^2\vec{r}^2 + \frac{4\pi\hbar^2(N-1)a}{m}|\Phi_{GP}(\vec{r})|^2\right] \times\Phi_{GP}(\vec{r}) = \epsilon_{GP}\Phi_{GP}(\vec{r})$$

- **GP + LHY corrections:**

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_{ho}^2\vec{r}^2 + \frac{4\pi\hbar^2(N-1)a}{m}|\Phi_{GP,mod}(\vec{r})|^2 \times\left(1+\frac{32}{3\sqrt{\pi}}a^{\frac{3}{2}}(N-1)^{\frac{1}{2}}\Phi_{GP,mod}(\vec{r})\right)\right]\Phi_{GP,mod}(\vec{r}) = \epsilon_{GP,mod}\Phi_{GP,mod}(\vec{r}).$$

Hardsphere Results for Trapped System



As in the homogeneous system, the inclusion of the quantum fluctuations or LHY term leads to greatly improved results.

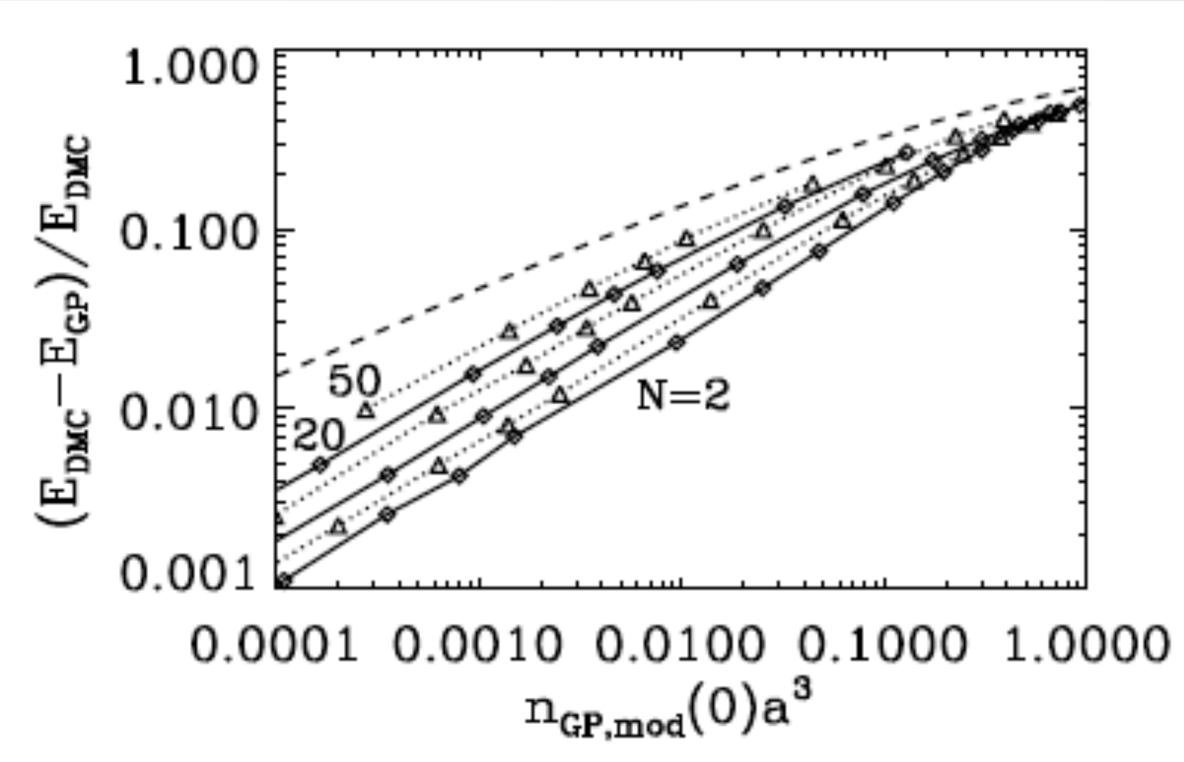
DMC for hardspheres higher than MF energy.

DMC for hardspheres lower/higher than modified MF energy.

Blume and Greene, PRA 63, 063601 (2001).

"Error" of GP Energy as a Function of Gas Parameter

Blume and Greene, PRA 63, 063601 (2001).



Dashed line: Estimate of beyond MF effects within TF approximation.

Summary: Equal-Mass Bosons are very Different from Equal-Mass Fermions

- Efimov physics in the universal regime where $a_s >> r_0$.
- Efimov's original proposal: Infinitely many three-body bound states.
- Current understanding: Low-energy properties of three- and four-body systems determined by a_s and a three-body parameter.
- Hard sphere Bose gas: Why we generally don't see signatures of three-body physics in dilute condensed Bose gases.
- Many open questions:

  B-F mixtures, unequal masses, five-body,…