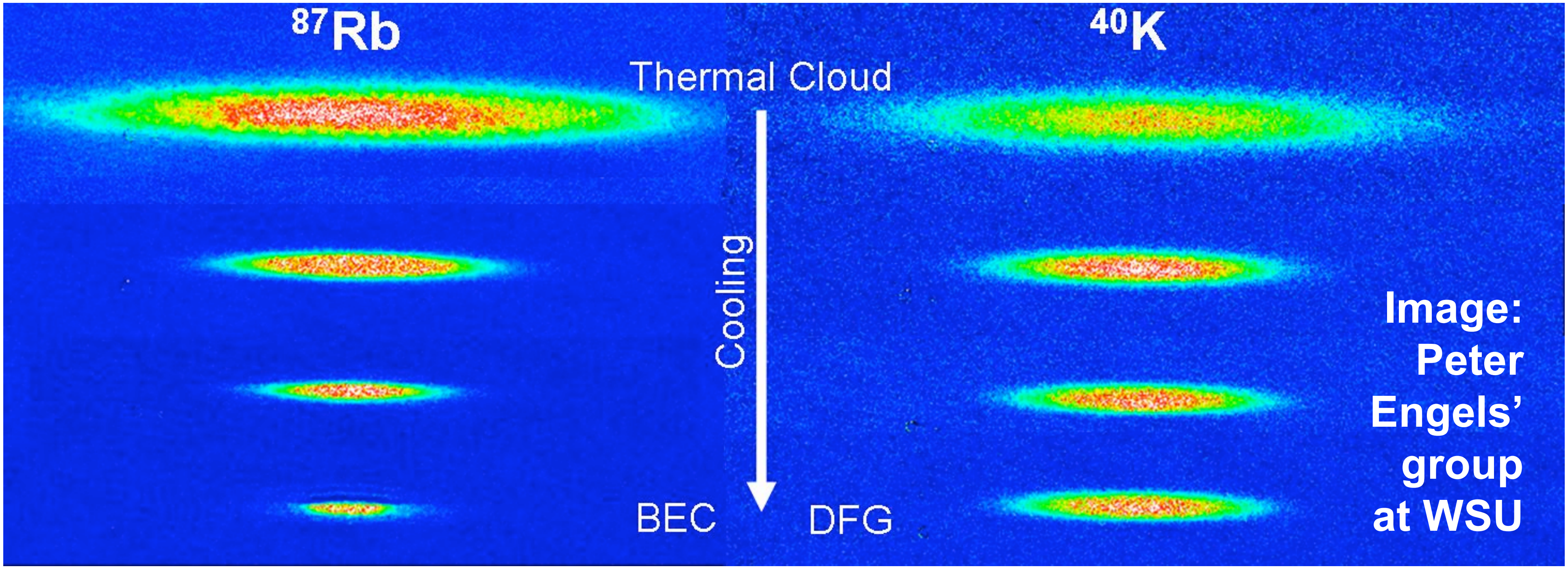

Unequal-mass Fermi Gases: Universal or Non-universal?

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At JILA: Chris Greene, Javier von Stecher, Seth Rittenhouse, John Bohn, Shai Ronen, Danielle Bortolotti.
At BEC Center in Trento: Stefano Giorgini, Grigori Astrakharchik.

Supported by NSF and ARO.

Outline of This Talk

- **Introduction:**
  - Why mass-imbalanced Fermi gases?
  - Open questions.
- **Main part:**
  - Crossover and BEC regime.
  - Results for three- and four-particle systems with infinitely large s-wave scattering length.
  - Homogeneous system.
- **Summary**

Why Fermi Mixtures With Unequal Masses?

- On the theory side:
  - Analogies with spin-imbalanced system.
  - At the mean-field level: “nothing new” (equations for equal masses rewritten in terms of reduced mass).
  - For mass ratios κ>13.607, three-body parameter is needed: Efimov physics.
  - Previous MC studies suggest existence of bound states or instability for κ<13.607: ZR versus FR?

- Experiments are gearing up:
  - Fermi-Fermi mixture: ^{40}K/^{6}Li ($\kappa\approx$6.7), ^{87}Sr/^{6}Li ($\kappa\approx$14.5), Yb/^{6}Li ($\kappa\approx$30), Yb/^{40}K ($\kappa\approx$4.5),…
  - Fermi-Bose mixture (one light particle): ^{87}Sr/^{7}Li ($\kappa\approx$12.4)

Why Fermi Mixtures With Unequal Masses? Specific Questions.

- Quantify finite-range effects:
  - Why?
    - Realistic atom-atom interactions have finite range.
    - Some numerical approaches only applicable to FR interactions.
  - How?
    - For N=3, comparison of energetics obtained for ZR interactions (analytical) and FR interactions (numerical).
    - For N>3, numerical: SV approach and Monte Carlo.

- In the (universal?) ZR limit:
  - Two-body versus three-body resonance.
  - How does three-body resonance appear?
  - Implications of three-body resonance for larger systems?

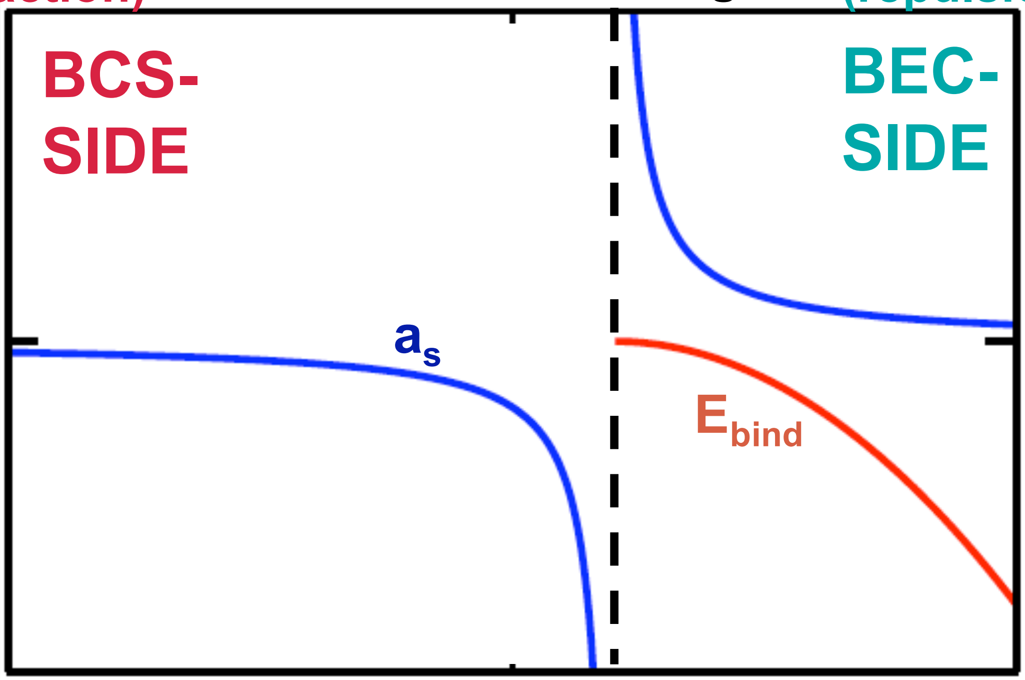
“Up/Heavy” - “Down/Light” Interaction: Two-Body s-Wave Scattering Length

weakly-interacting (attraction)

strongly-interacting

weakly-interacting (repulsion)

$$a_s = \lim_{k \to 0} -\frac{\tan(\delta_s)}{k}$$



external control parameter (B-field)

Dilute gas:

$r_0 \ll a_{ho}$, a_s or

$n(0)r_0^3 \ll 1$.

We find: Need to go to $r_0 \sim 0.001 a_{ho}$ in certain regimes.

For unequal masses: Use reduced mass in scattering problem.

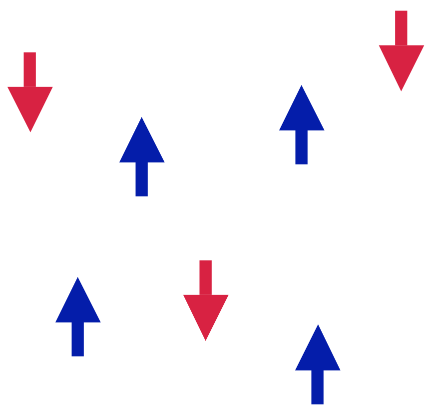
BCS-BEC Crossover with Cold Two-Component Atomic Fermi Gas: $m_1=m_2$

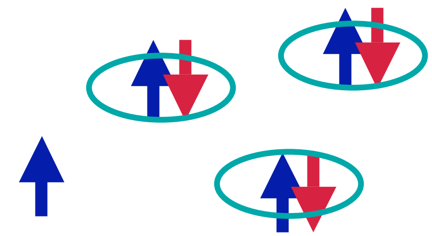
Weakly-attractive atomic Fermi gas

"BCS" — "BEC" — a_{ho}/a_s

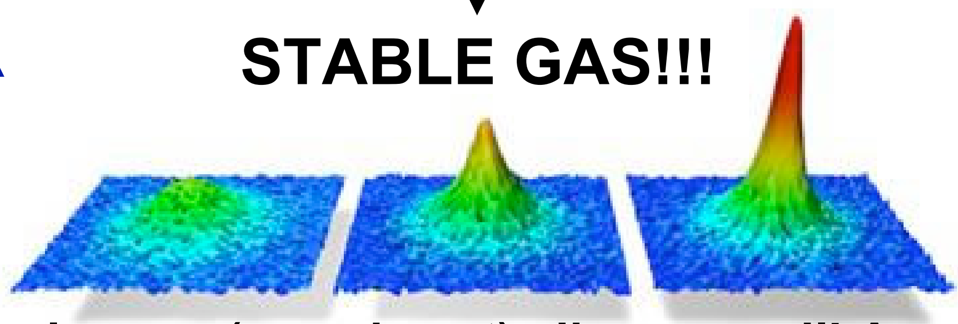
Strongly-interacting (unitarity)

Weakly-repulsive molecular Bose gas





STABLE GAS!!!



Images (experiment): Jin group, JILA.

No bound trimers or tetramers.
$a_{ad}=1.12a_s$.
$a_{dd}=0.6a_s$.

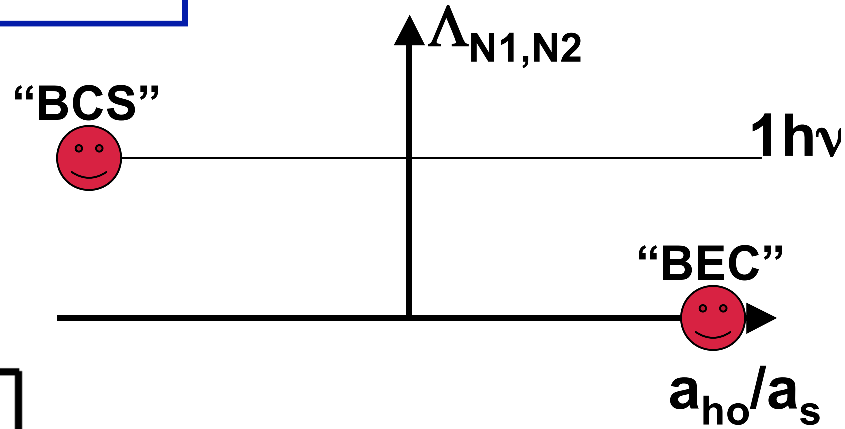
Crossover determined by a_s alone (one two-body parameter).
Unitary and NI regime have same number of length scales.
For unequal masses: Do trimers or tetramers exist?
Do novel many-body phases exist?

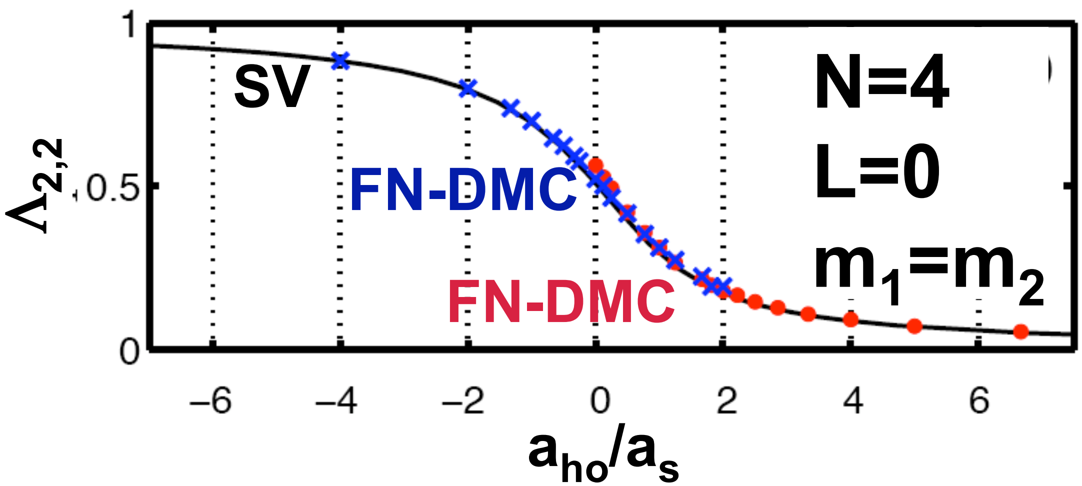
Universal Energy Crossover Curve for Trapped Fermi System

Universal energy crossover curve:

$$\Lambda_{N_1,N_2} = \frac{E(N_1,N_2) - N_d E(1,1) - 3N_f \hbar\omega/2}{E_{NI} - 3N\hbar\omega/2}$$

N_f: Number of unpaired fermions.
N_d: Number of dimers formed.

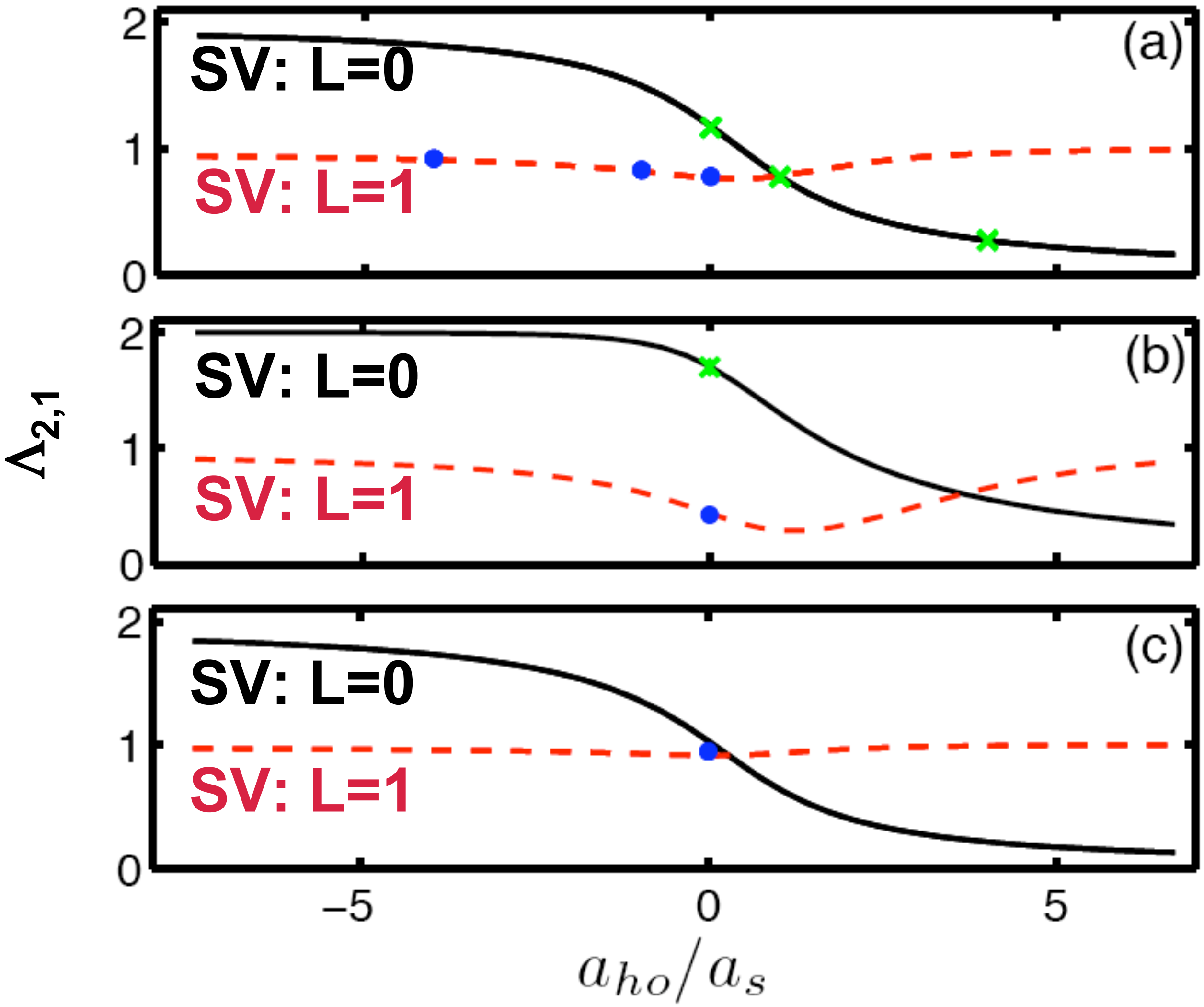




For example, four fermions per site in deep optical lattice.

von Stecher, Greene, Blume, PRA 76, 053613 (2007).

N=3: Energy Crossover Curve for Mass Ratios 1 and 4 (Equal Frequencies)



Equal masses

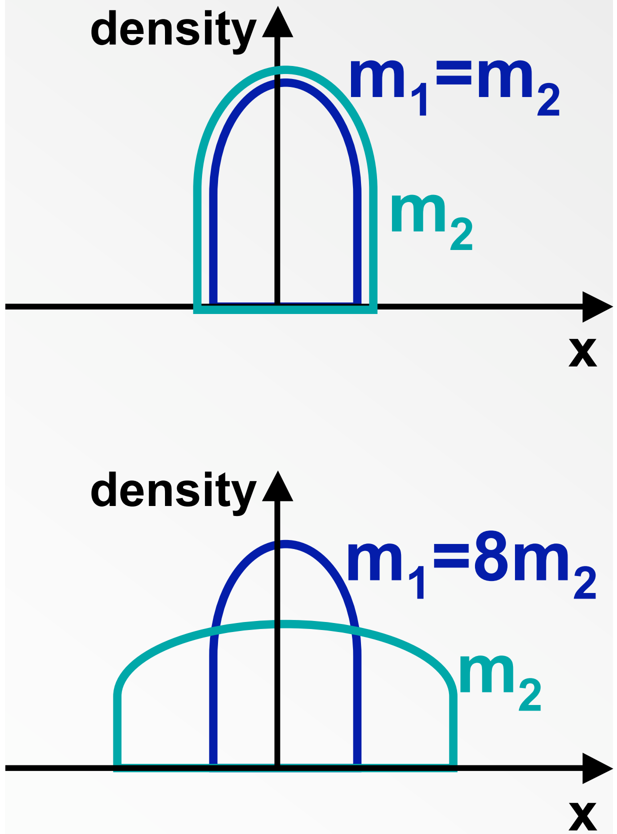
Two heavy, one light

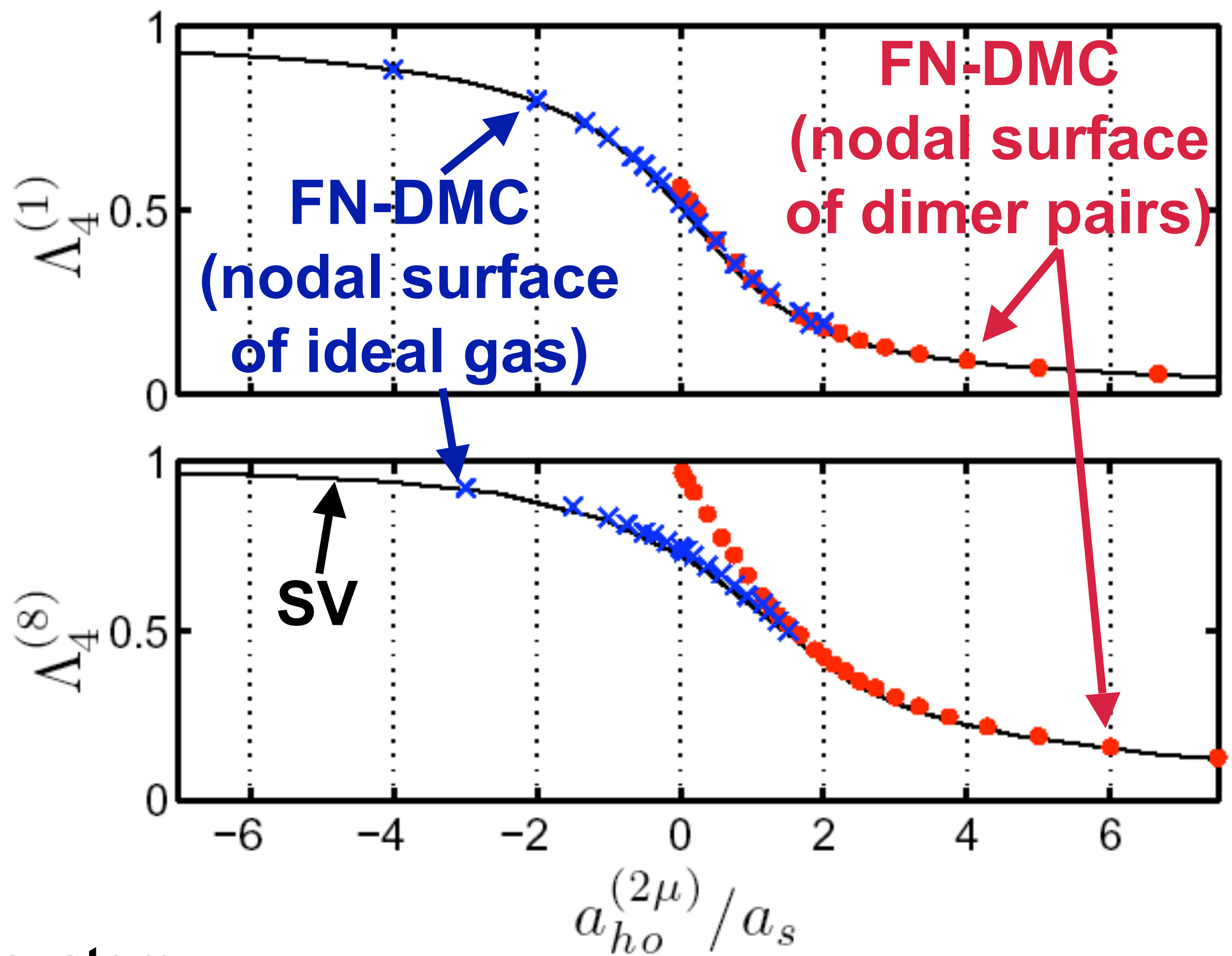
One heavy, two light

Symbols: FN-DMC with different guiding functions.

von Stecher, Greene, Blume, PRA 77, 043619 (2008).

Crossover Curve for N=4: Different Mass Ratios, Equal Frequencies





Size of non-interacting system:

$$a_{ho}^{(j)} = \sqrt{\hbar/(m_j\omega_j)}$$

von Stecher, Greene, Blume, PRA 76, 053613 (2007).

Determination of Dimer-Dimer Scattering Length and Effective Range

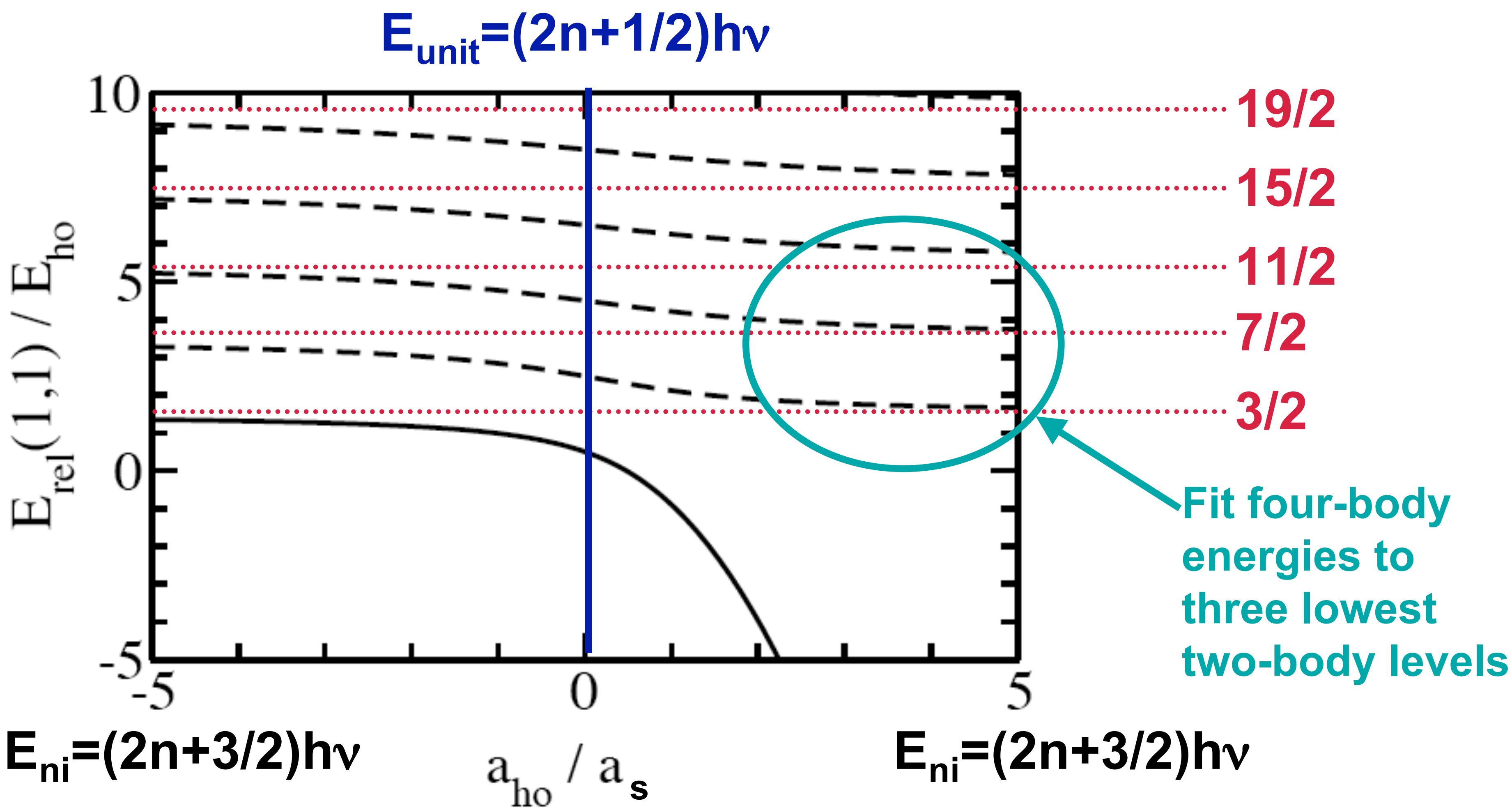
- Perform full scattering calculation:
  - Analytical treatment: ZR interactions (Petrov).
  - Numerical treatment: FR interactions within hyperspherical framework (Greene group).

- Our idea: Perform bound state calculations for four-fermion system and, assuming formation of two composite bosonic dimers, extract dimer-dimer phase shift through comparison with analytically known two-particle solution for ZR interactions.

- Analog of Luescher's formula, which is often used in nuclear theory (two particles in a box: relationship between phase shift and eigenenergy).
- Extremely important for lattice QCD calculations.

M. Luescher, Nucl. Phys. B 354, 531 (1991); 364, 237 (1991).

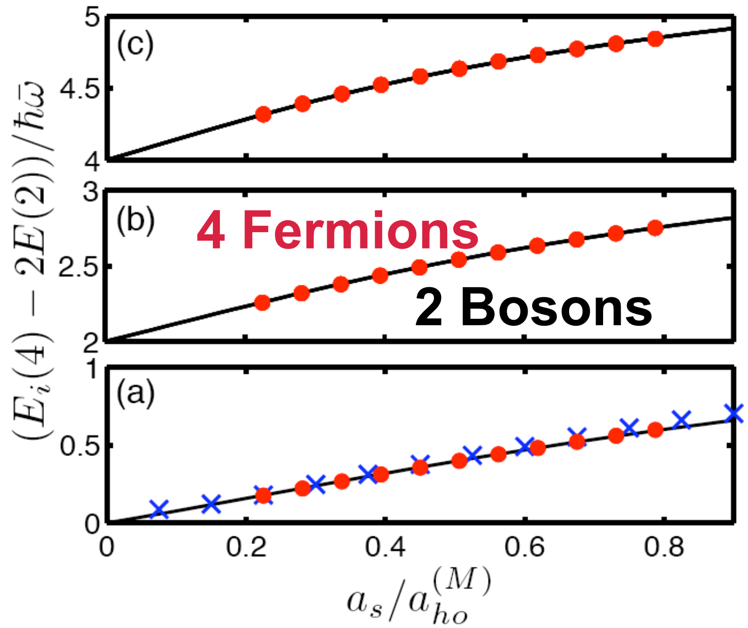
Two s-Wave Interacting Particles in External Spherically Harmonic Trap



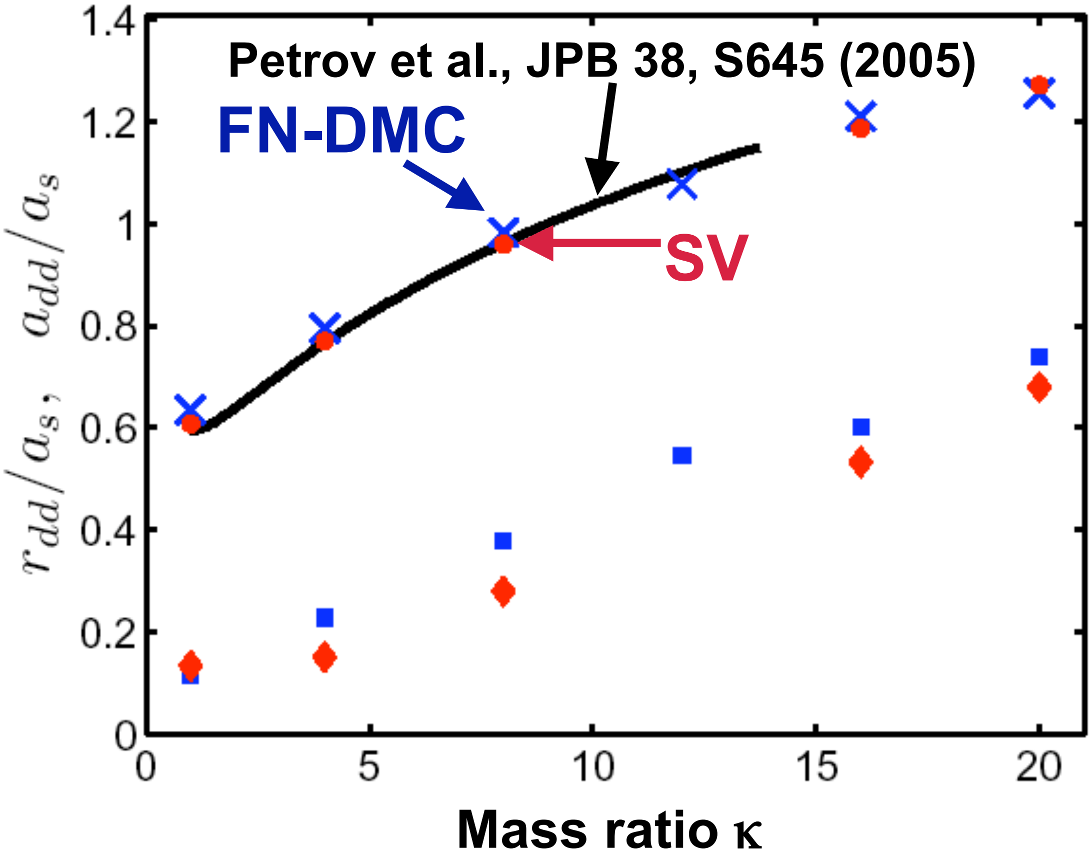
Analytical treatment: Busch et al., Found. of Phys. (1998).

Dimer-Dimer Scattering Length and Effective Range (4 Fermions=2 Bosons)

von Stecher, Greene, Blume, PRA 76, 053613 (2007).



First quantitative prediction for dimer-dimer effective range.



For large mass ratio, we consider dimer-dimer branch; other branches exist...

r_{dd} increases with increasing mass ratio κ: For κ=20, r_{dd}~$0.5a_{dd}$.

For κ>13.607, see also B. Marcelis et al., PRA 77, 032707 (2008).

From now on, Unitarity: Hyperspherical Treatment of Trimer

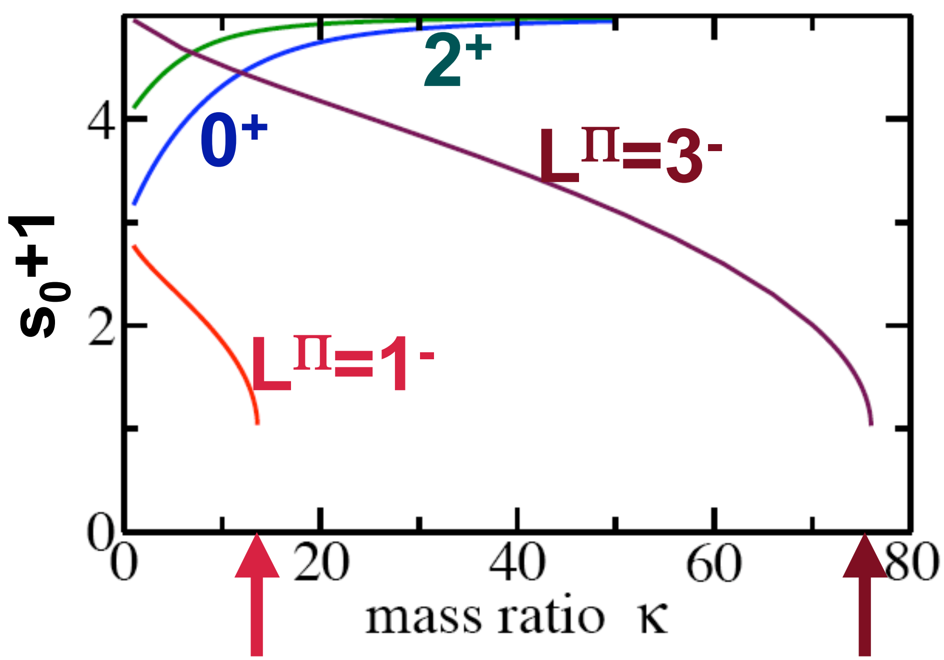
- Exact separability.
- Relative (internal) wave function: $\Psi_{rel}(R,\Omega)=R^{-5/2}F(R)\Phi(\Omega)$.
- Solution to hyperangular wave function gives ν:

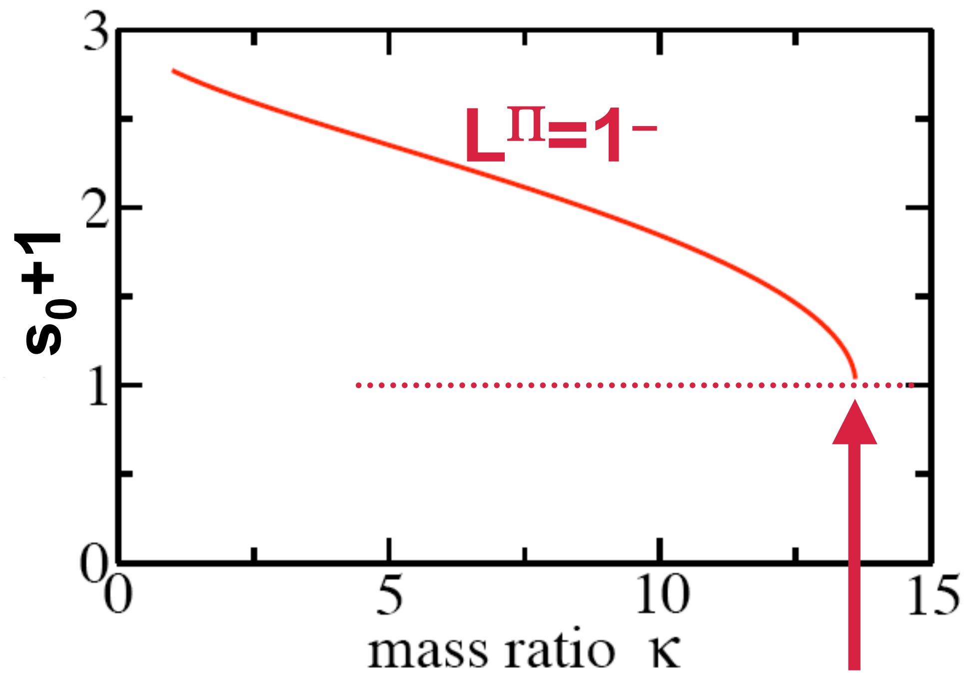
$$H_R = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial R^2} + \frac{\hbar^2\nu(\nu+1)}{2MR^2} + \frac{1}{2}M\omega^2R^2$$

- Defining $\nu = s-1/2$ gives: $\nu(\nu+1) = s^2-1/4$.
- For the three-body problem, s is known for all symmetries, angular momenta and mass ratios.
- If we take s as given, then problem reduces to solving 1D differential equation in effective hyperradial potential (in this case, simple analytical form).

Hyperangular Solution (2 Heavy, 1 Light; ZR Interactions with $a_s^{-1}=0$)

Analytical solution for s [Efimov (1971); see also, e.g., Kartavtsev and Malykh, JPB 40, 1429 (2007); Rittenhouse, Ph.D. thesis, CU Boulder (2009)]:



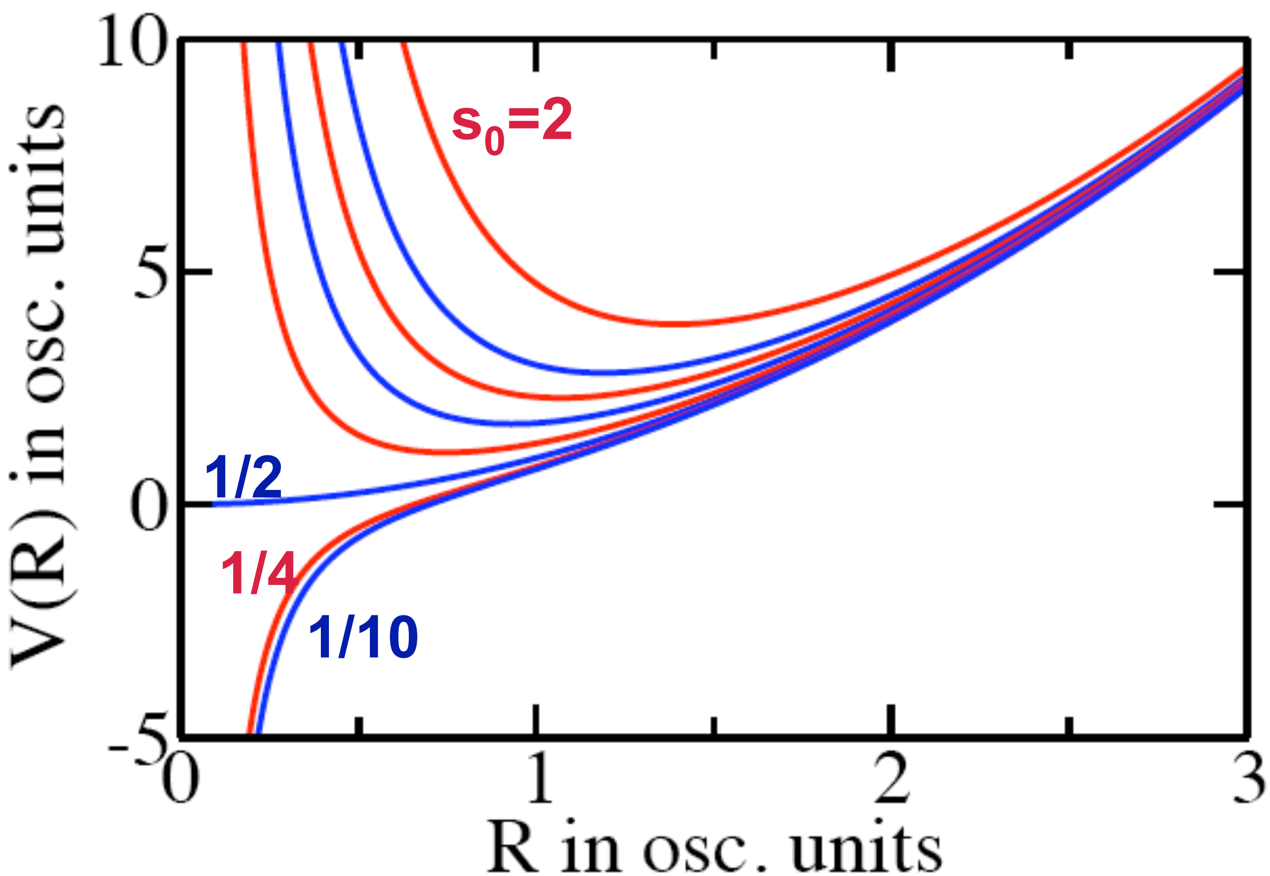


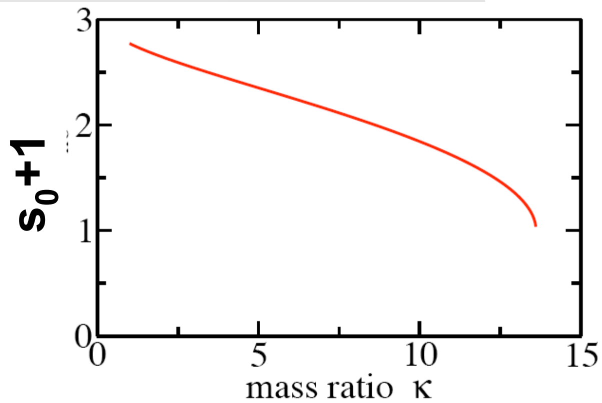
Beyond these mass ratios, s_0 becomes purely imaginary.
Three-body parameter is needed → Efimov physics (for L=1 state, 13.607…).

L=1, a_s>0 and κ>8.172…: 1 three-body bound state exists in free space.

Effective Hyperradial Three-Particle Potential Curves For $L^{\Pi}=1^{-}$

$$V(R) = \hbar^2(s^2-1/4)/(2\mu R^2)+\mu\omega^2R^2/2$$





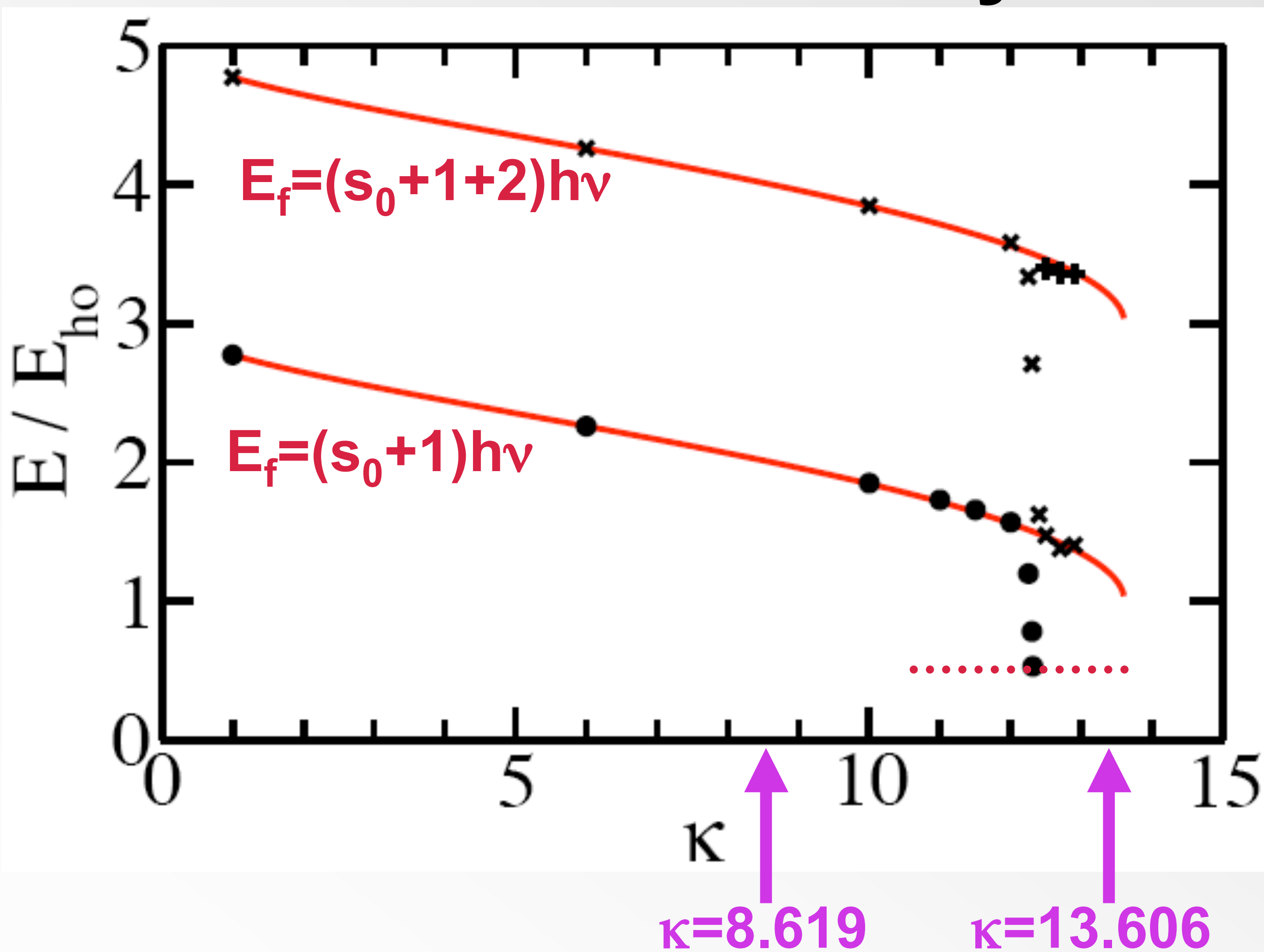
For κ=12.3131, we have s_0=1/2: Just the usual isotropic HO?

Hyperradial Solution (s_0=1/2 Example): What to Keep and What to Eliminate?

- Want to solve: $[-1/2F''(x) + x^2/2\, F(x)] = \varepsilon F(x)$; $x=R/a_{ho}$, $\varepsilon=E/h\nu$.
- Two linearly independent solutions (no BC specified yet):
  - $f(x) = A\, x \exp[-x^2/2]\ {}_1F_1(-\varepsilon/2+3/4,\ 3/2,\ x^2)$
  - $g(x) = B \exp[-x^2/2]\ {}_1F_1(-\varepsilon/2+1/4,\ 1/2,\ x^2)$
- Write: $F(x) = f(x) \cos(\pi\mu) - g(x) \sin(\pi\mu)$.
- Take large x limit and constrain μ such that $\exp(x^2/2)$ pieces cancel.
- Consider small x limit and impose BC: $F'(0)/F(0) = -a_{ho}/b$.
- Of course: For NI isotropic HO, we have F(0)=0 and b=0.
- However: In our case, b cannot be determined within ZR framework (b depends, in general, on two-body potential).
- If $F(x) = f(x)$: $E = E_f = (2n+3/2)h\nu = (2n+s_0+1)h\nu$.
- If $F(x) = g(x)$: $E = E_g = (2n+1/2)h\nu = (2n+s_0)h\nu$.

Borca, Blume, Greene, NJP 5 (2003).

Trapped Fermi-System (2 heavy, 1light) with $L^{\Pi}=1^{-}$ at Unitarity: ZR vs. FR



Black symbols: SV energies for FR Gaussian extrapolated to $r_0=0$.

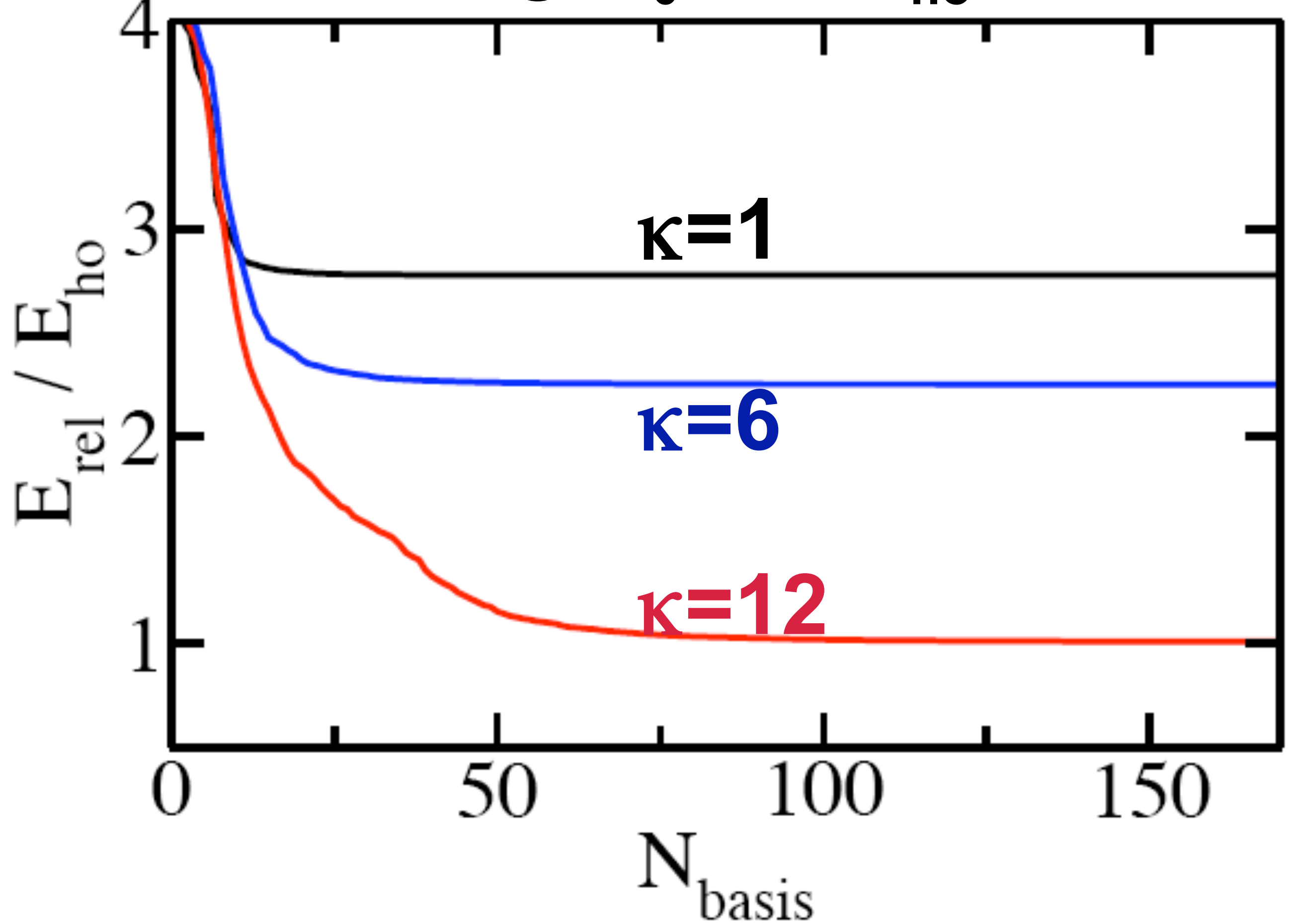
Away from κ=12.3131, extrapolated FR energies agree with E_f.

κ=12.3131, extrapolated FR energy agrees with $E_g=h\nu/2$.

The possibility of having a "three-body resonance" for 8.169< κ<13.606 (regular and irregular solution contribute) was pointed out by Nishida et al., PRL 100, 090405 (2008).

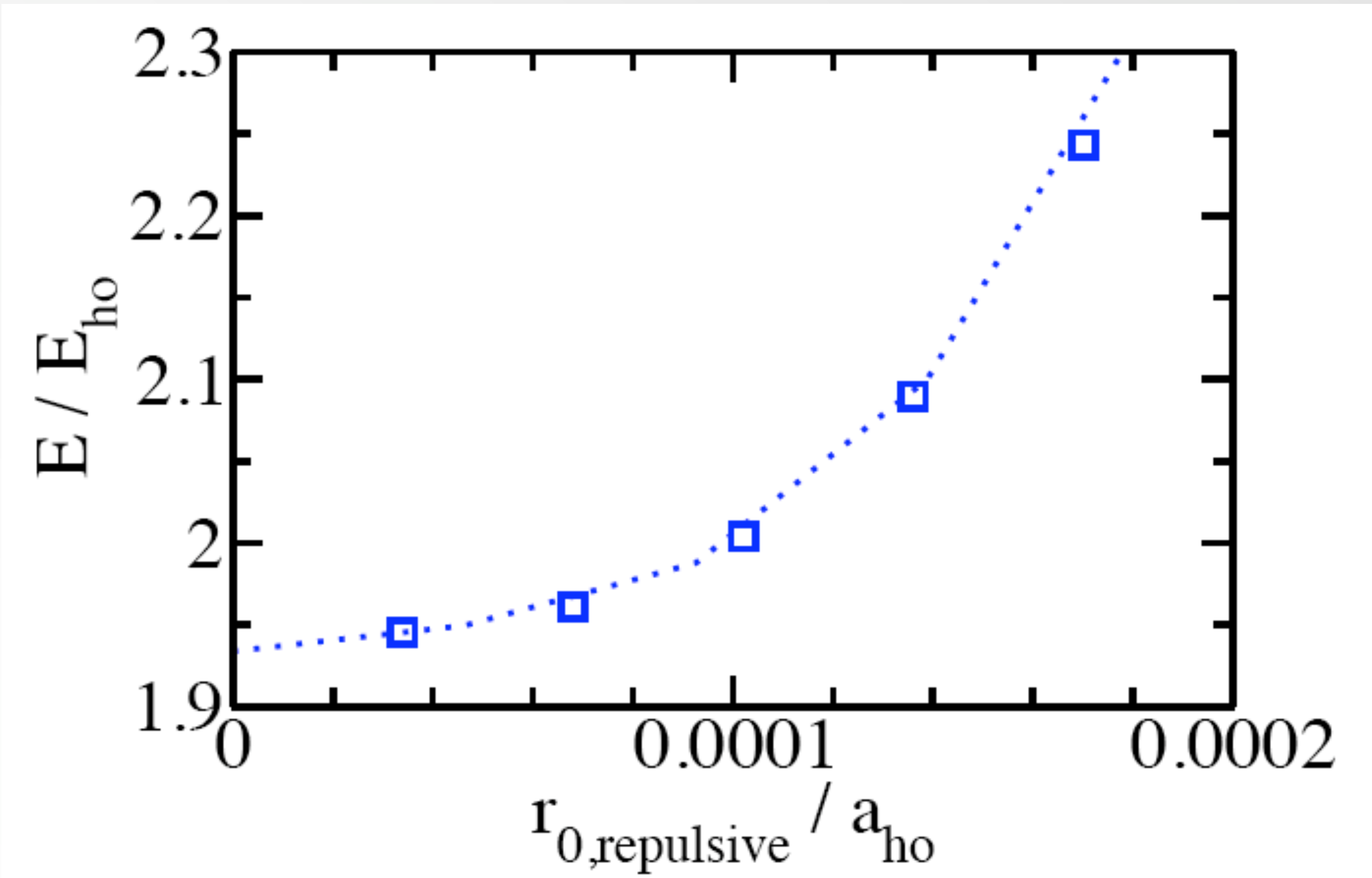
It's not Numerics…
Convergence for Different Mass Ratios

range r_0~$0.01a_{ho}$



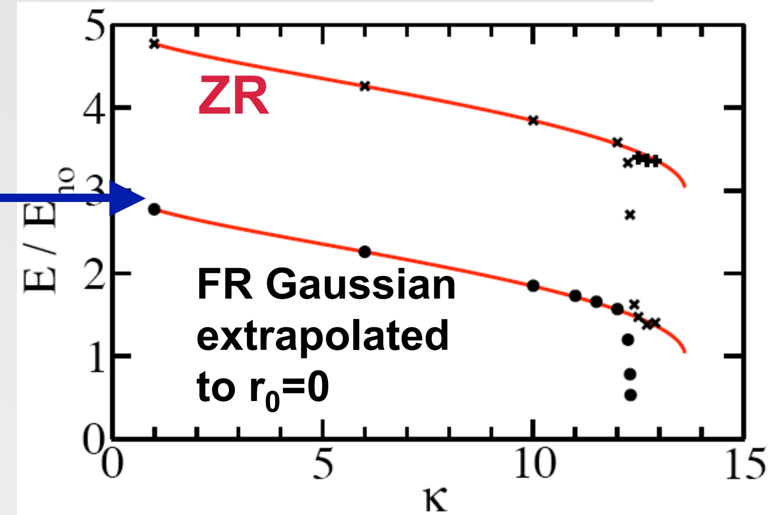
It's not Numerics: Add SR Repulsive Piece…

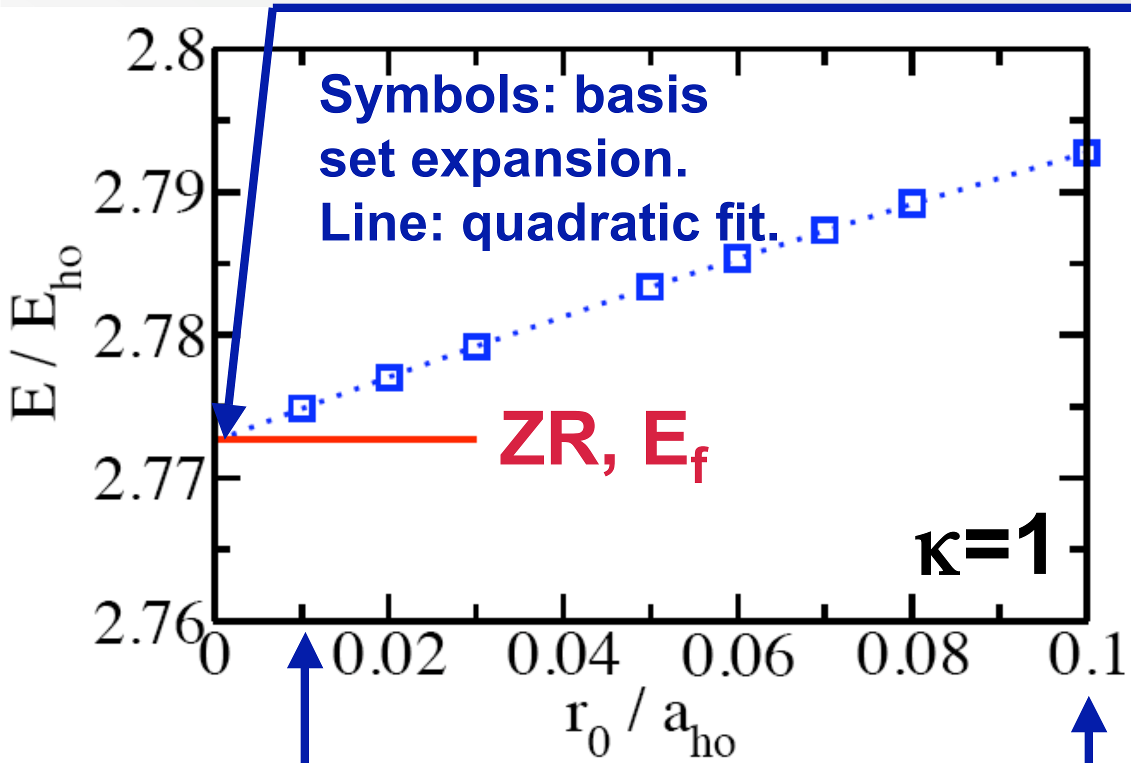
Mass ratio 12.5, L=1, first excited state with r_0~$0.0025a_{ho}$.



Stable results!

Trapped Fermi-System (2 heavy,1 light) with $L^{\Pi}=1^{-}$ at Unitarity: ZR vs. FR



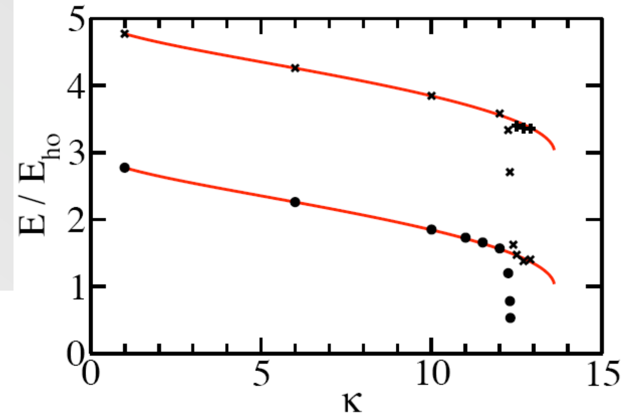


interaction range/ trap length=10^{-2}

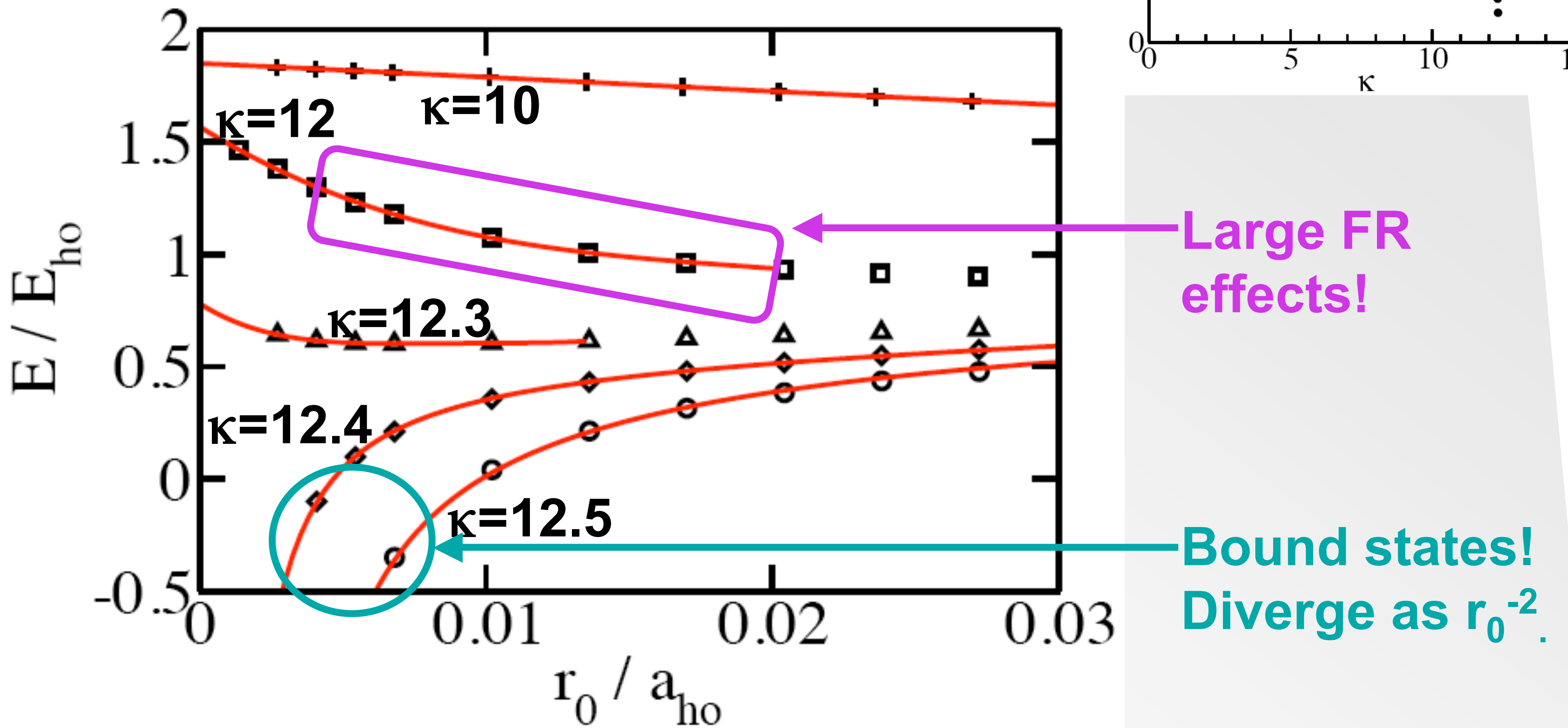
interaction range/ trap length=10^{-1}

Finite value of interaction range to trap length is dealt with straightforwardly.

Trapped Fermi-System (2 heavy,1 light) with $L^{\Pi}=1^-$ at Unitarity: ZR vs. FR



Symbols: SV energies.
Red lines: Four to five parameter fits.

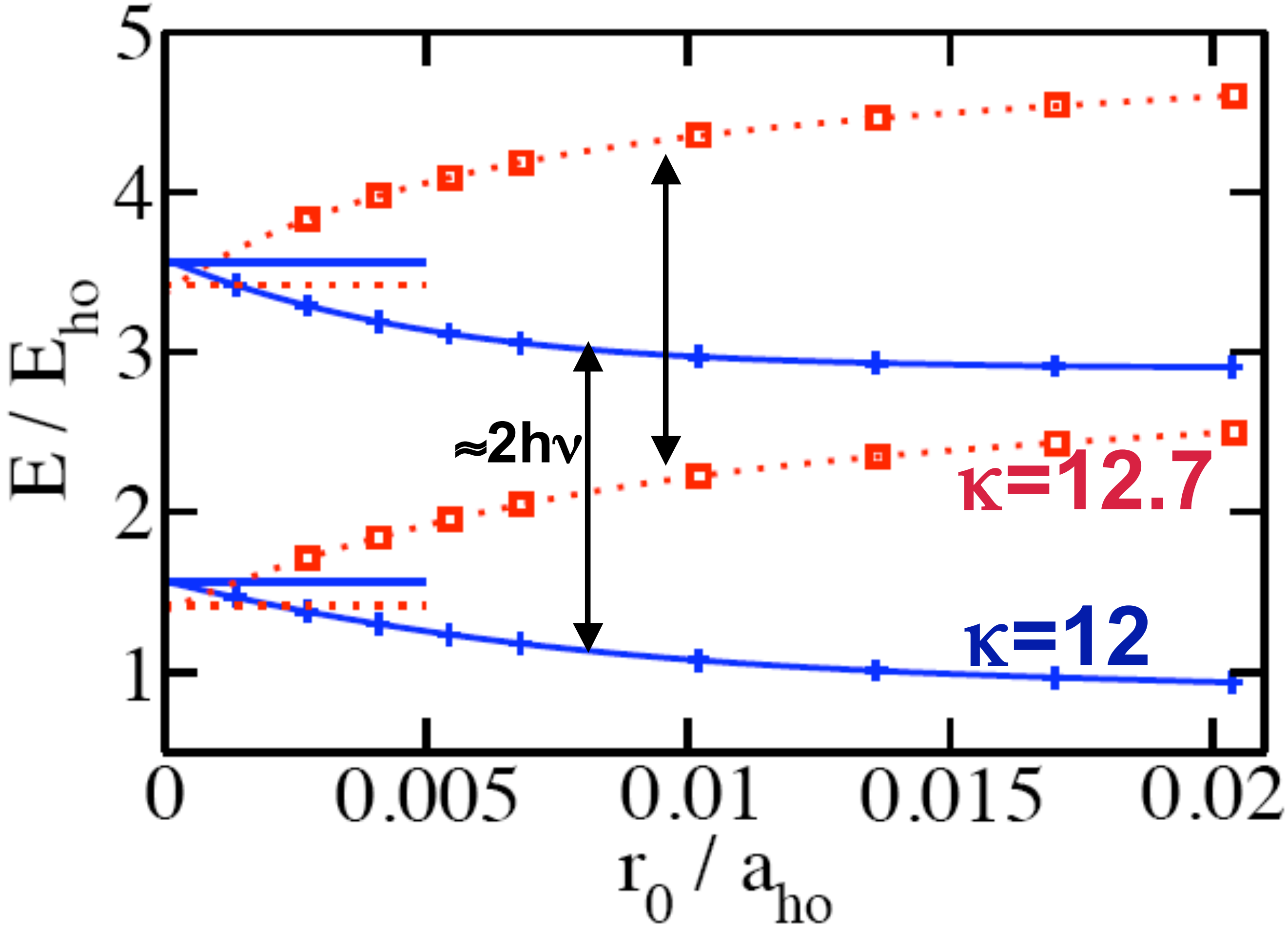


Trapped Fermi-System (2 heavy,1 light) with $L^{\Pi}=1^{-}$ at Unitarity: ZR vs. FR

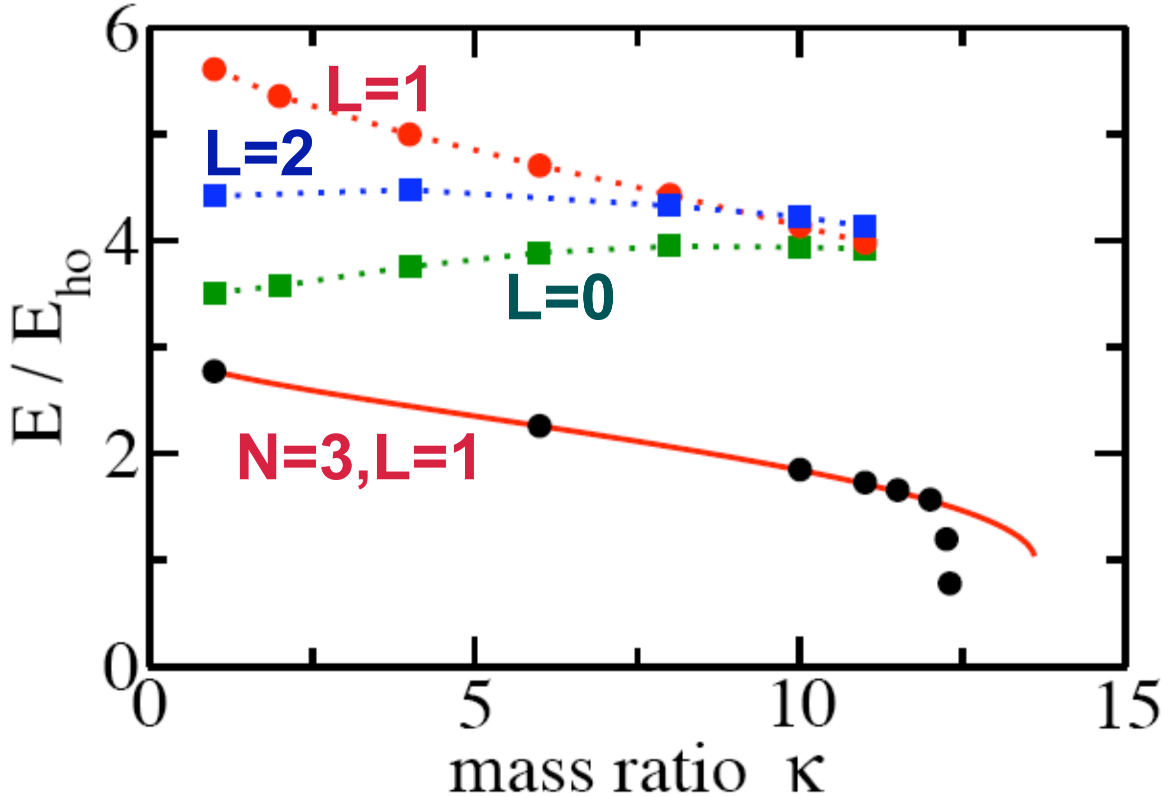
Symbols: SV.

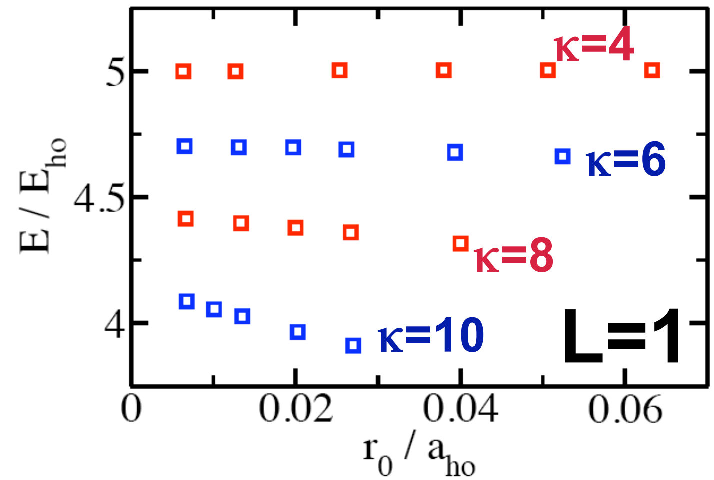
Lines: fit.

Horizontal lines: ZR energy E_f.



Four-Particle System (2 Heavy, 2 Light) at Unitarity



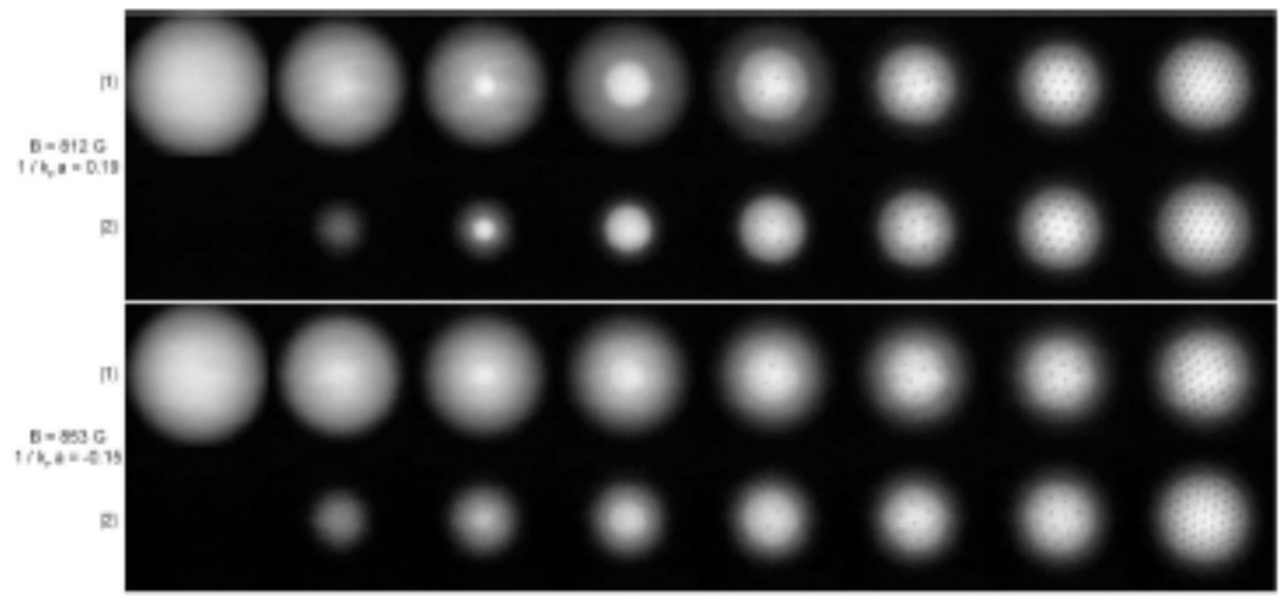


N=4, L=1 energies nearly "parallel" to N=3, L=1 energies:
Indicative of universal physics?

N=4, L=0 and 2: No negative energy states up to κ=11.

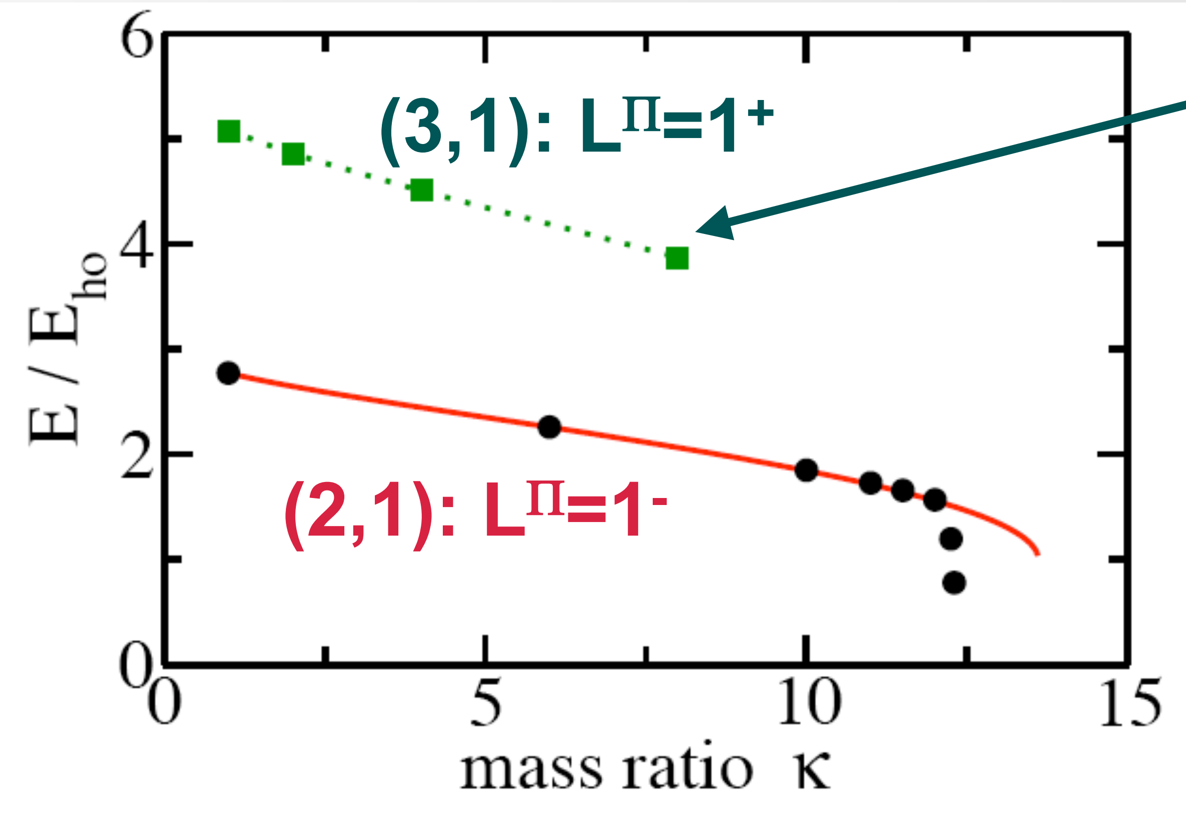
Polarized System: 3+1 and 4+1

- Whole range of interesting physics: polaron physics, phase separation, etc. (see MIT and Rice experiments).
- Ground state of 3+1 and 4+1 system have unnatural parity [$\Pi=-(-1)^L$]: Need more general basis functions in SV approach.
- κ=1: Comparison between SV and FN-DMC energies.



Zwierlein et al.,
Science 311, 492 (2006).

Preliminary Results: Four-Particle System (3 heavy,1 light) with $a_s^{-1}=0$



FR energies approach ZR limit from below: Extrapolated energy will likely go up.

For comparison, Joe Carlson and coworkers find (unpublished):
(2,1): $\kappa_{cr}=13.6$
(3,1): $\kappa_{cr}=10.5$
(4,1): $\kappa_{cr}=9.5$

Implications of Few-Body Results for Unequal Masses at Unitarity

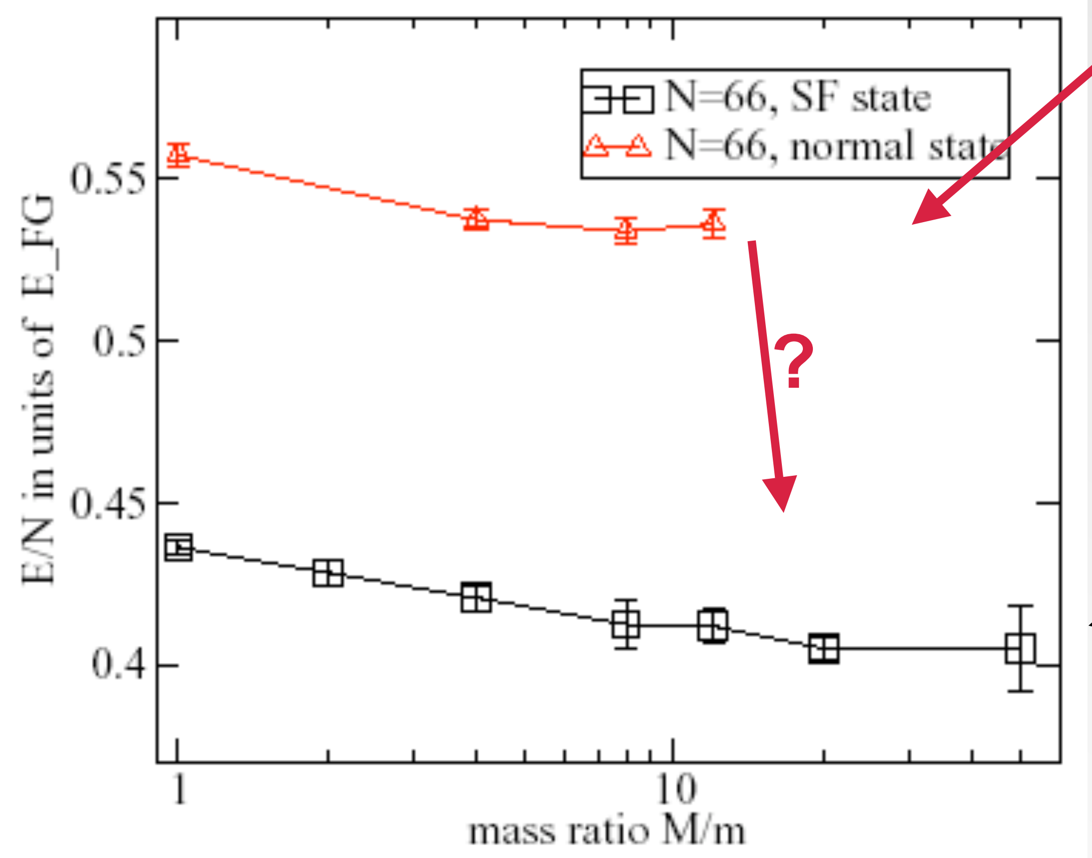
- Finite range effects surprisingly large - had been partially overlooked by some of our earlier studies at unitarity and in BEC regime.
- E.g., need to double-check effective range of dimer-dimer system.

- Three-body resonance realized *exactly* at s_0=1/2 (κ=12.3131)?

- Implications for homogeneous system:
  - Can some of previously unresolved results be explained by FR effects?
  - How to reach ZR limit?

E/N for Homogeneous System at Unitarity: "SF versus Normal State"

Unpublished FN-DMC results for $N_M=N_m$ by Astrakharchik, Blume and Giorgini ('06/'07); $nr_0^3=10^{-6}$:



Stability of normal state? Threebody or cluster states?

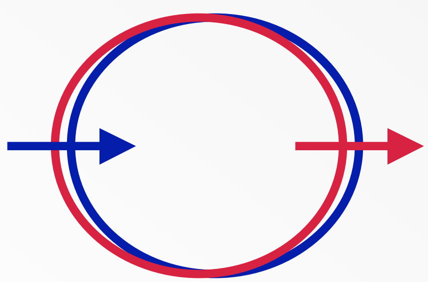
State compatible with superfluidity has lower energy than normal state.

Mass ratio of 50! SF state appears "numerically stable"!

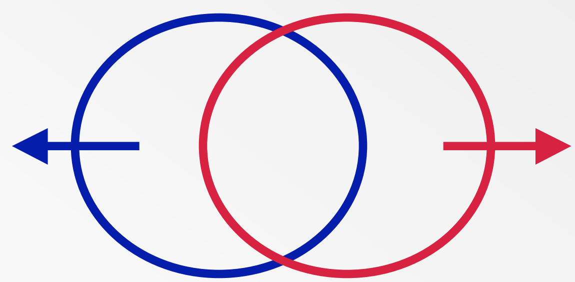
See also results by Gezerlis et al., PRL 103, 060403 (2009); Joe Carlson's talk at INT, Seattle, in March 2010.

Energetics Suggest (But Need to Determine Compressibility Matrix…)

- Small M/m: Normal state is energetically less favorable than state compatible with superfluidity.
- Nodal surface of normal state seems compatible with “cluster formation”: indication of an instability.
- Nodal surface of state compatible with superfluidity seems to prohibit “cluster formation”; this state is “numerically” more stable than normal state.



In-phase oscillation:
Allowed in NF and SF.



Out-of-phase oscillation:
Allowed in NF but not in SF.

Excitation gap gives system rigidity with respect to spin oscillations?

Outlook:
What's Next on the To Do List?

- Solve three-body problem with ZR and FR interactions as a function of scattering length for different mass ratios.
- Determine structural properties of the three-body bound state.
- Go to larger systems (4+1 and 3+2) to check if absence of negative energy states holds up to mass ratio of about 12.
- Estimate lifetime of excited three-body state.
- Go back to many-body system… SF state stable because it effectively excludes three-body correlations?

Summary and Implications: Unequal-Mass Systems are Rich and Non-Trivial

- Finite-range effects increase with increasing κ (become tremendous).
- L=1 three-body bound states exist for κ>12.3131 for a class of short-range model interactions at unitarity.
- How to treat many-body system?
  - FN-DMC calculations (as currently implemented) cannot go to sufficiently small r_0 for large κ. Can/should the ground state be eliminated for κ>12.3131?
  - Can FN-DMC calculations map out range-dependence for small κ?
  - Treat ZR interactions directly (do not extrapolate)?