Elastic transmission of atoms through ^{4}He films

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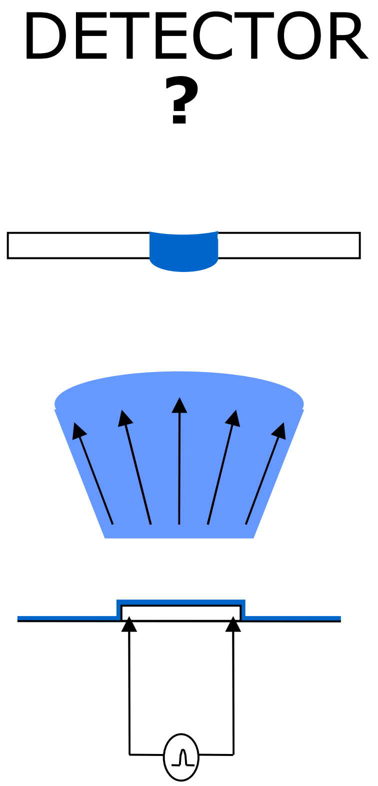
Transmission experiment

It is now possible to suspend films of He-II.

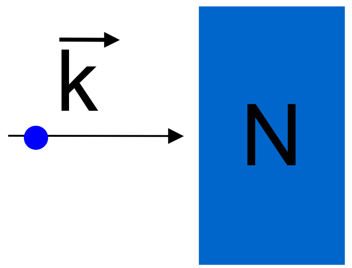
A beam of He atoms is shot unto a free-standing film of He-II. Transmitted particles are detected.

Such a measurement provides a probe for the structure of superfluid.

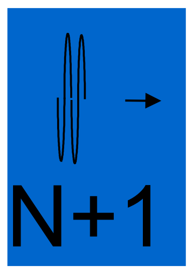
(The only probe currently is neutron scattering – which works at energies much greater than characteristic for He-II).



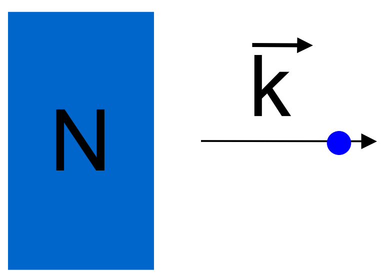
Quasiparticle mediated transmission



Initial state:
quantum condensation.



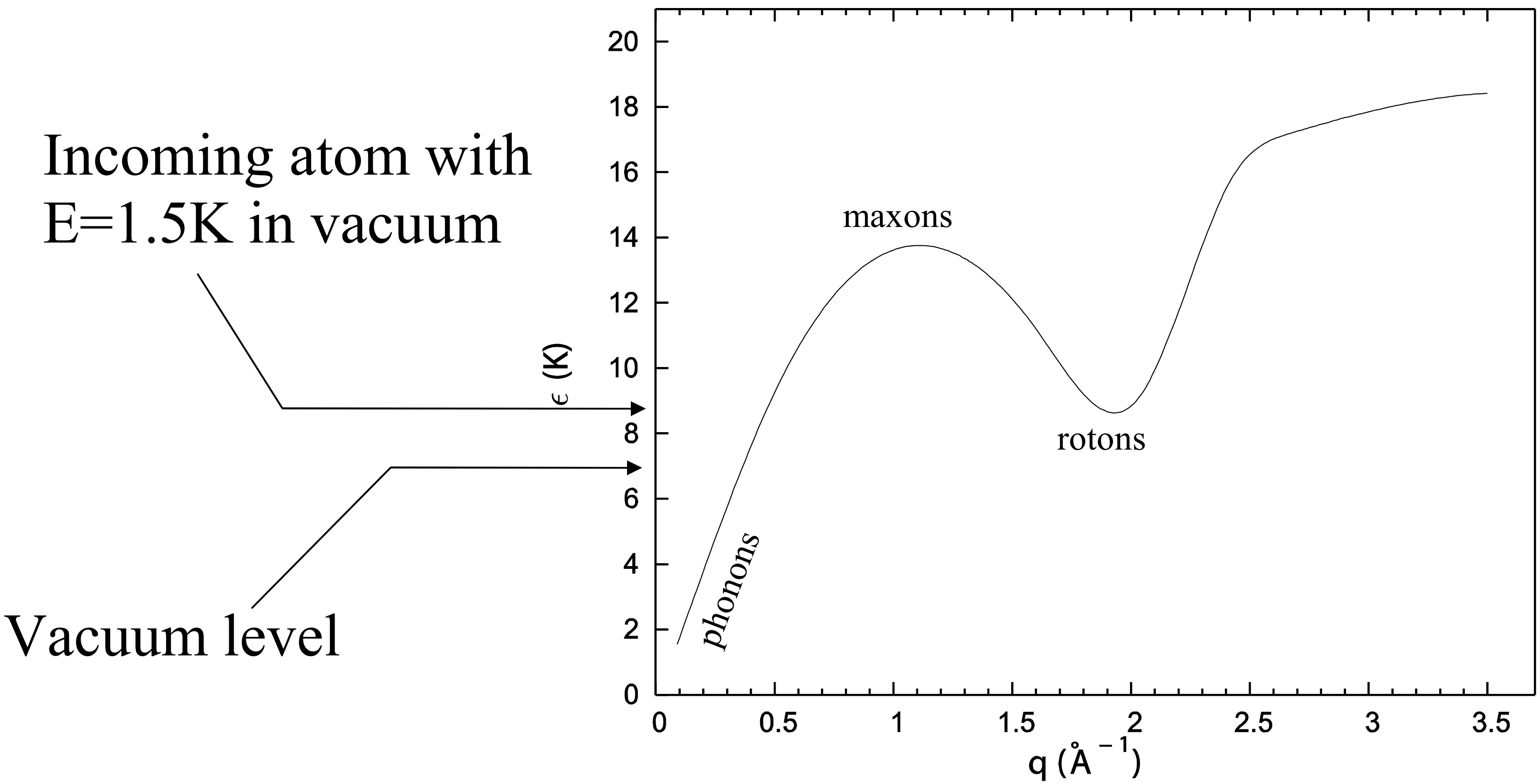
Intermediate state: propagating
quasiparticle (real).



Final state: reemission
by quantum evaporation.

^{4}He excitation spectrum

He-4 atoms have chemical potential of -7.2K in helium-II.



Quasiparticle mediated transmission

Transmission is facilitated by quasiparticles in the fluid.

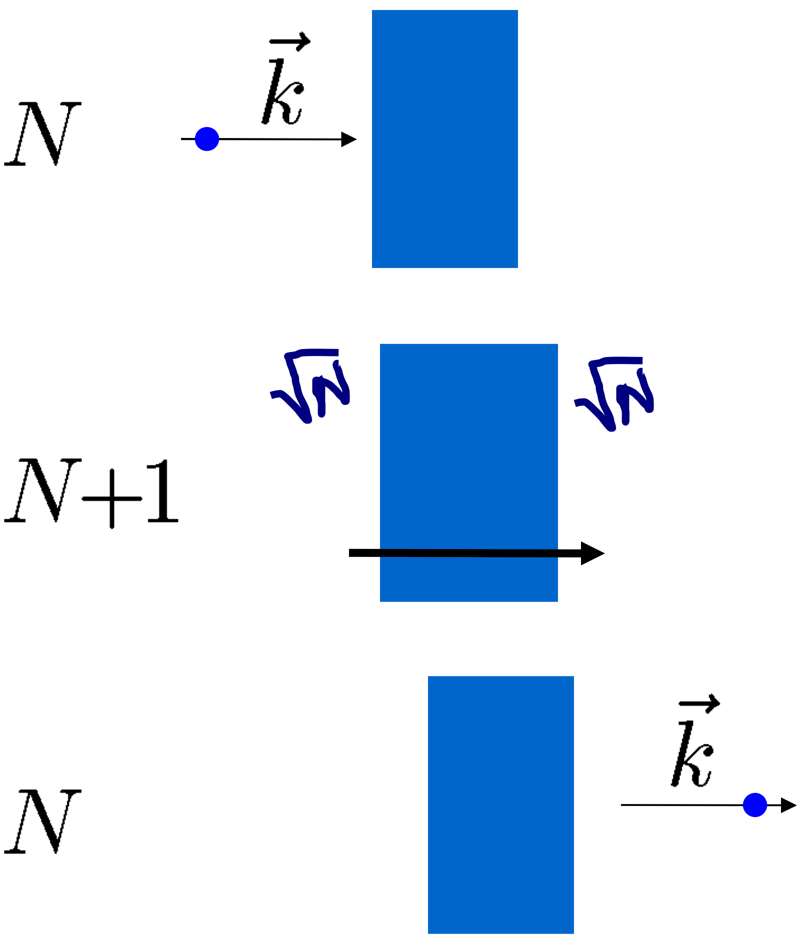
Quasiparticles are created via quantum condensation, expel atoms via quantum evaporation.

Expect transmission by phonons [1] and rotons [2]. Both channels were observed.

Real process, characterized by QP propagation through the film.

[1] JLTP 140, 429 (2005)   [2] PRL 91, 085301 (2003)

Condensate mediated transmission



Initial state

$$E_N + \epsilon_k$$

Intermediate *virtual* state

$$E_N + \epsilon_k + \mu$$

Final state: reemission

$$\Delta E \Delta t \approx \hbar \qquad \Delta E \approx |\mu|$$

Condensate mediated transmission

Transmission may be mediated by exchange of the incoming particles with the condensate. This may be a possible probe into the condensate structure.

The intermediate state is virtual and consists of the slab *and* the incoming particle in the ground state in the moving slab.

Transmission times are estimated by the uncertainty principle: Times are independent of the slab thickness and much shorter than those for quasiparticle-assisted transmission. This may facilitate detection.

Monte Carlo calculations

CBF calculations predict a large transmission amplitude for a wide range of momenta of incoming incoming atoms. Near-elastic transmission was observed experimentally. Up to 15% of all atoms are transmitted elastically to within experimental precision.

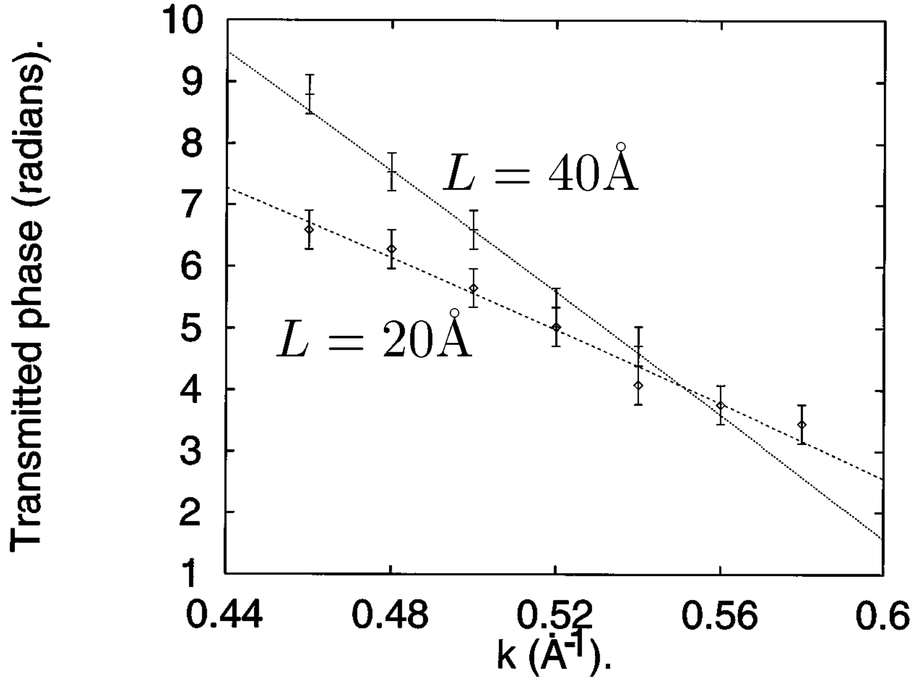
We only consider elastic processes, with the goal of testing different elastic transmission channels.

[PRL 80, 2169 (1998)]

Variational calculations

$$\Psi_T = \sum_l \phi_{inc}(r_l)\,(1-\nu_{env}(r_l))\prod_{i\neq l}\nu_{env}(r_i)\Psi_J^{N+1}(r_1,\cdots,r_{N+1}) + c.m.m.$$

$$J = const, \quad E = E_N + \frac{\hbar^2 k^2}{2m}$$



[PRL 79, 3930 (1997)]

Diffusion Monte Carlo

$$f(\tau) = \Psi_T \exp\left[-(\mathcal{H} - E_{ref})\tau\right]\Psi_{IC} \to \Psi_T A_0 \exp\left[-(E_0 - E_{ref})\tau\right]\phi_0 \propto \phi_0$$

$$\frac{\partial f}{\partial \tau} = \sum_i^N \frac{\hbar^2}{2m_i}\left[\nabla_i^2 f - \nabla_i \cdot (f\vec{U}_i)\right] - \left(\frac{\mathcal{H}\Psi_T}{\Psi_T} - E_{ref}\right) f$$

Diffusion (random walk).

Drift with velocity field

Displace points with $\vec{U}_i \tau$

$$\vec{U}_i = \nabla_i \log |\Psi_T|^2$$

Local change in *f* (create or destroy points)

Scattering states for the elastic transmission problem.

Non-zero boundary as one but only one of the particles is removed far from the slab:

$$z_1, \ldots, z_{N+1} \to z_1, \ldots, z_{i-1}, \pm\infty, z_{i+1}, \ldots, z_{N+1}$$

Set boundary conditions with the scattering states:

$$\Psi(r_1, \ldots, r_{N+1}) \to \Psi_k^{SC}(r_i)\Psi(r_1, \ldots, r_{i-1}, r_{i+1}, \ldots, r_{N+1}),\ z_i \to \pm\infty$$

For VMC and Bogoliubov approximation calculations, these could simply describe left-to-right propagation:

$$\Psi_k^R(z_i) = T_k e^{ikz_i},\ \ \Psi_k^L(z_i) = e^{ikz_i} + R_k e^{-ikz_i}$$

This is insufficient for DMC as the complex-valued scattering states cannot be used to fix boundary conditions via the guiding function. Need real-valued scattering states.

Scattering states for the elastic transmission problem.

It is possible to rewrite the scattering states in terms of real-valued standing waves (we only need two, corresponding to possible symmetries under reflection of all particles about z=0). These states map onto plain wave scattering states.

$$\Psi_k^+ = \cos(kz - \delta_e \mathrm{sign} z) = \cos(k|z| - \delta_e)$$

$$\Psi_k^- = \sin(kz + \delta_o \mathrm{sign} z) = \mathrm{sign}(z) \sin(k|z| + \delta_o)$$

$$\Psi(z) = \mathrm{e}^{i\delta_e} \Psi_k^+(z) + i\mathrm{e}^{i\delta_o} \Psi_k^-(z)$$

$$R_k = \frac{1}{2}\left(\mathrm{e}^{2i\delta_e} - \mathrm{e}^{2i\delta_o}\right)$$

$$T_k = \frac{1}{2}\left(\mathrm{e}^{2i\delta_e} + \mathrm{e}^{2i\delta_o}\right)$$

[PRA 67, 013610 (2003)]

Determining the scattering phases

Correct phase is determined by the energy condition:

$$E = E_N + \hbar^2 k^2 / 2m$$

Approach: Perform simulations at different fixed phase shifts, separately for odd and even symmetry. Determine correct phases for each wavelength in this manner.

$$\Psi_T(r_1, \ldots . r_{N+1}) = \left[\sum_i^{N+1} \Psi_k^{\pm}(z_i)(1 - \nu(z_i^*)) \prod_{j \neq i} \nu(z_j^*) \right] \Psi_J^{N+1}(r_1, \ldots, r_{N+1})$$

$$\nu(z) = \frac{1}{(1 + \exp(\frac{z-a}{b}))(1 + \exp(\frac{-z-a}{b}))} \qquad z_i^* = z_i - z_{c.m.}$$

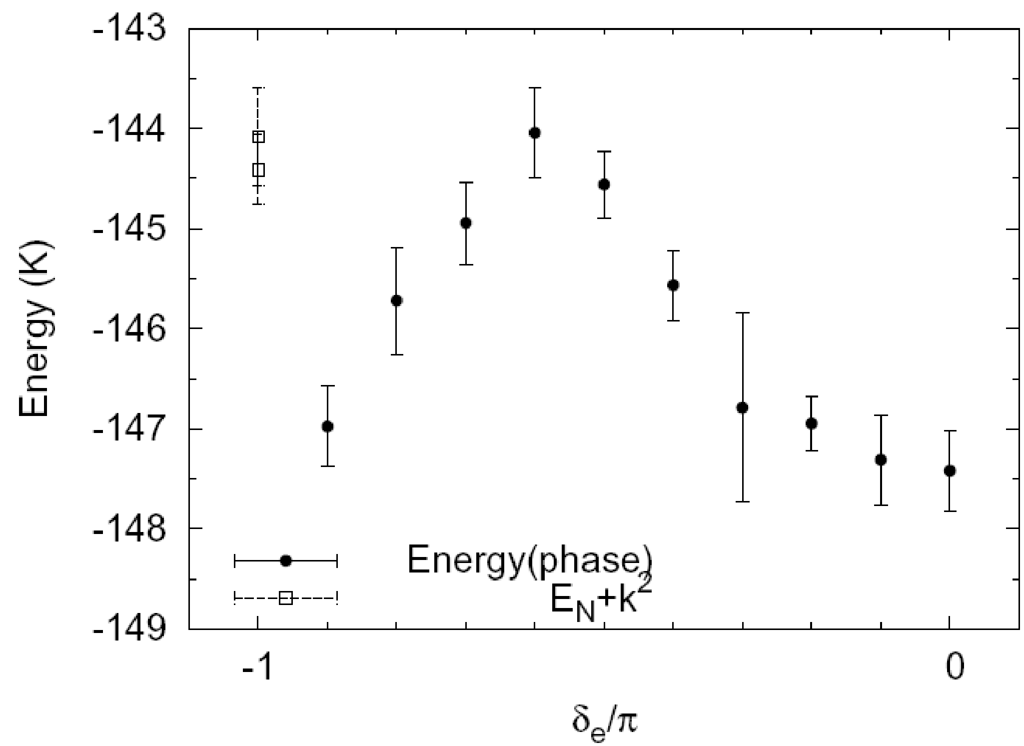
$$\Psi_J^{N+1}(r_i, \ldots, r_{N+1}) = \prod_{i<j} \exp\left(-\frac{1}{2} \left(\frac{s}{r} \right)^5 \right)$$

Determining the scattering phases

32 atoms in a long, narrow channel, periodic in transverse direction.

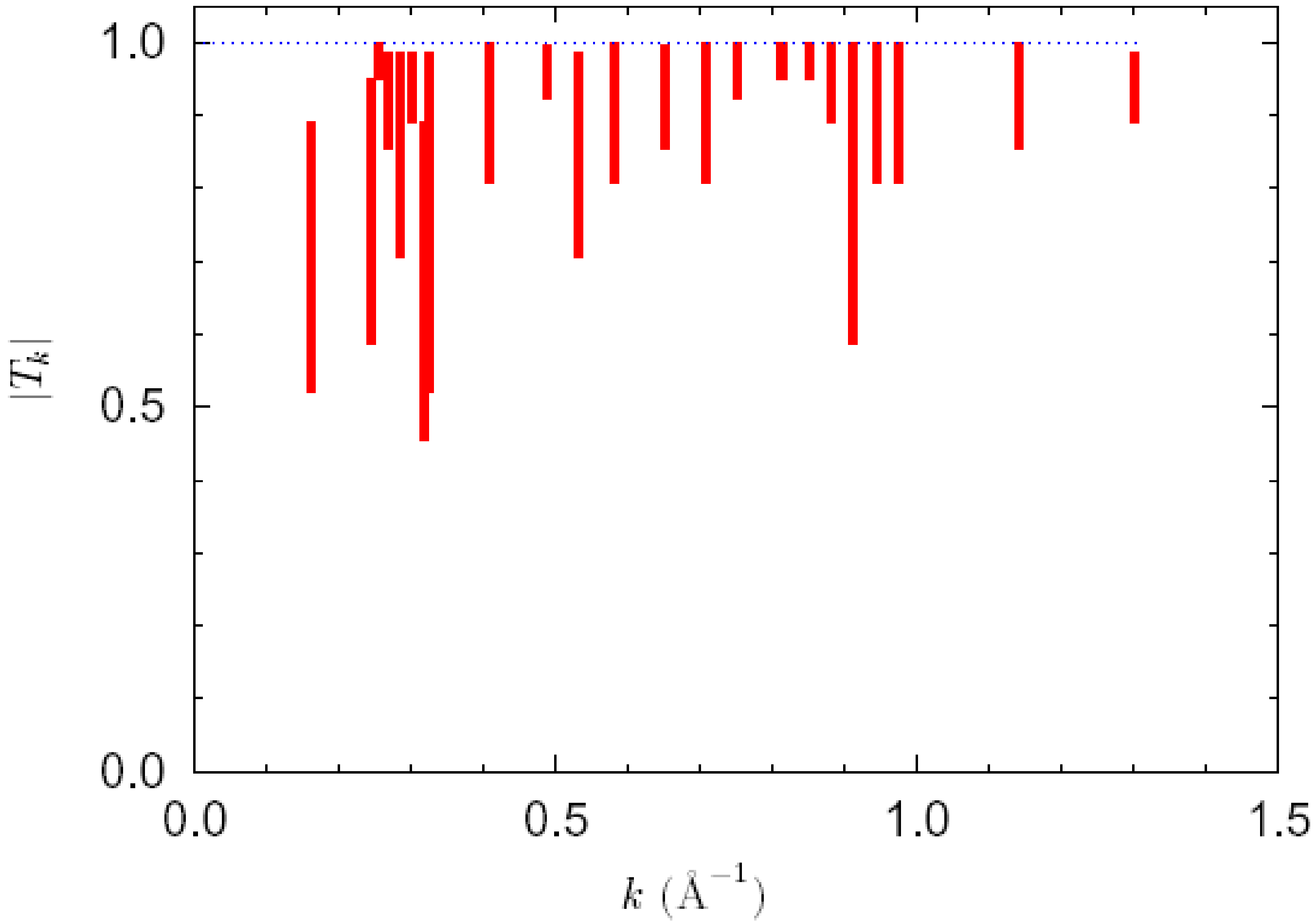
About 50 Å thick, 6.2x6.2Å in transverse direction.

50 K^{-1} projection in imaginary time in ½ million steps.



Multiple calculations which is computationally demanding. Greatly improved with second order (DMC2a) decomposition.

Transmission coefficient



Absolute value of the transmission coefficient vs. the wavevector of the incoming particle.

Transmission time analysis

Propagation of a particle after it leaves the slab can be described in terms of a wavepacket. Arrival of the peak of the packet at a detector can be found from stationary phase argument. This comes to

$$t_{\rm slab} - t_{\rm noslab} = \frac{1}{v_k}\frac{d\phi}{dk} \qquad T_k = |T_k|\,{\rm e}^{{\rm i}\phi(k)}$$

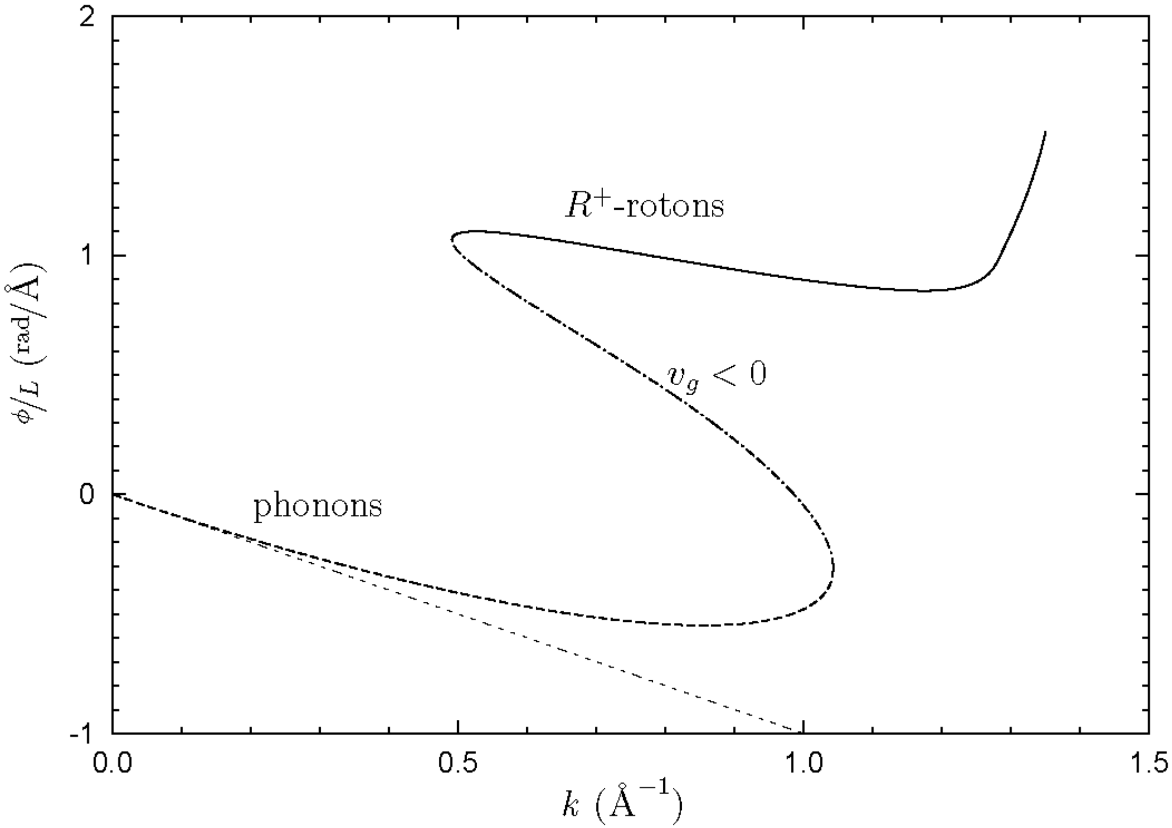
For the condensate mediated transmission, the transmission times are expected to be negligible:

$$\phi = \phi_0 - L(k - k_0)$$

For quasiparticle mediated transmission with a single quasiparticle mode:

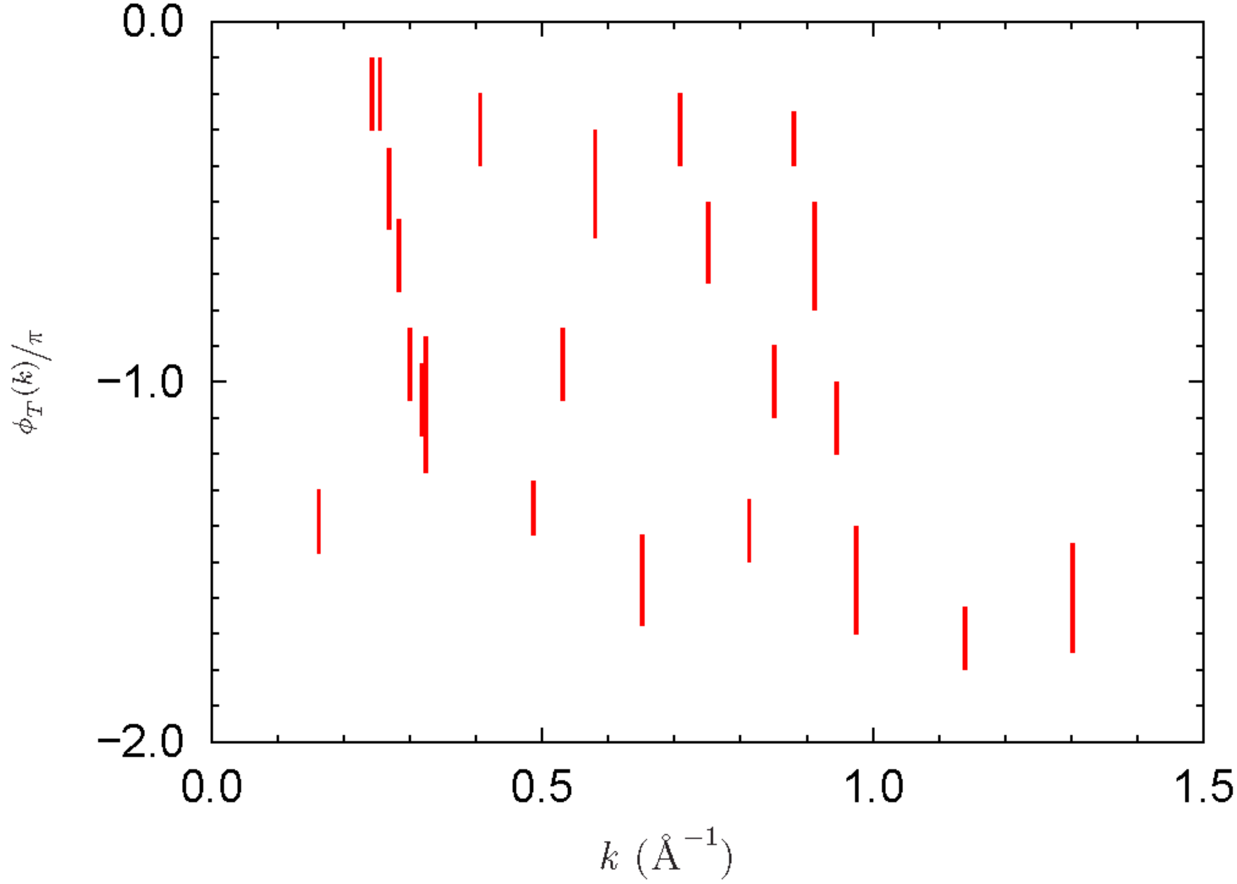
$$\phi - \phi_0 = L\left[-(k - k_0) + (q_k - q_{k_0})\right]$$

Transmission time analysis



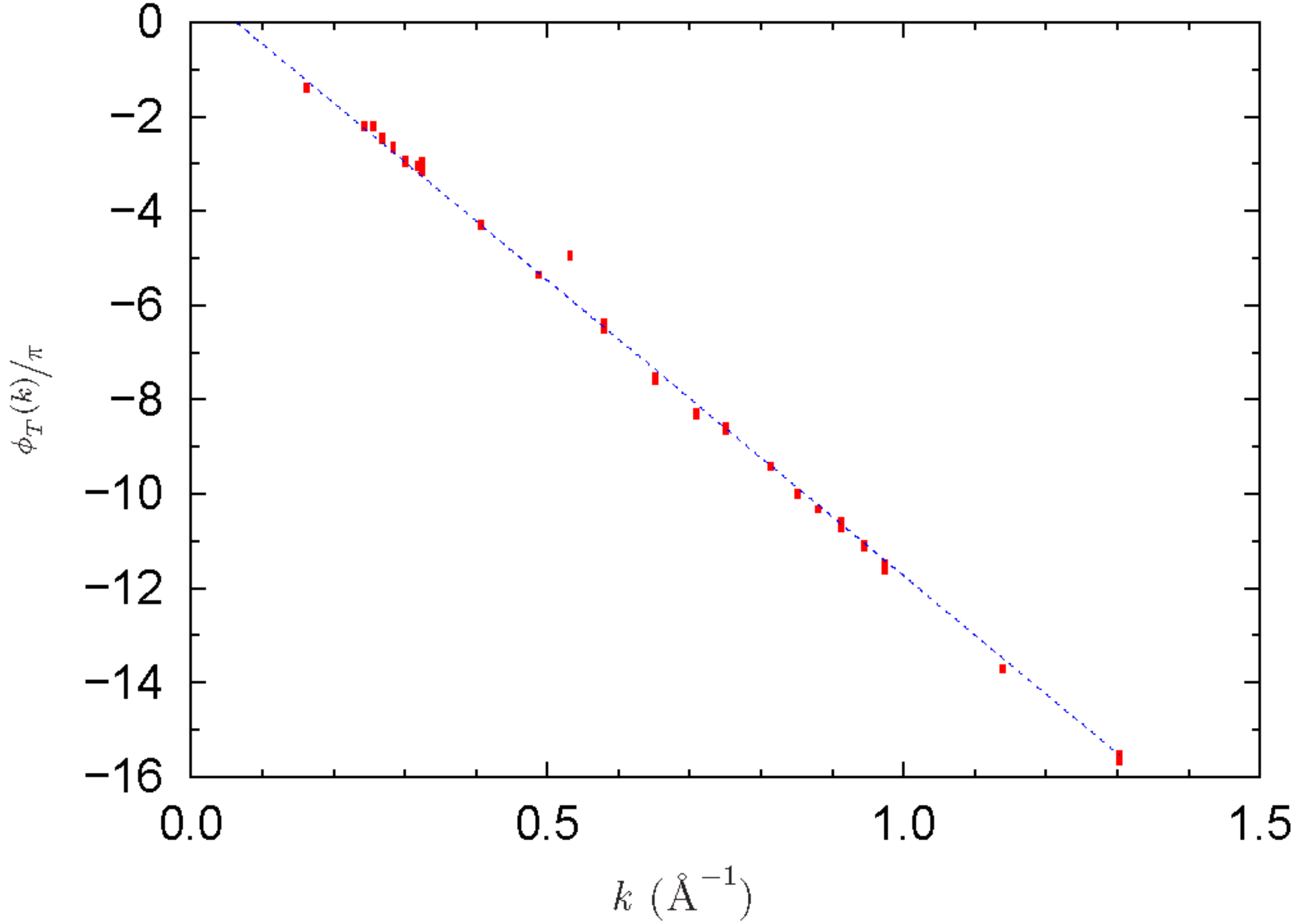
We can derive the model phase dependence for individual scattering channels. Actual phase may be a result of superposition of different channels, which may be fitted in analysis to extract amplitudes.

Phase results



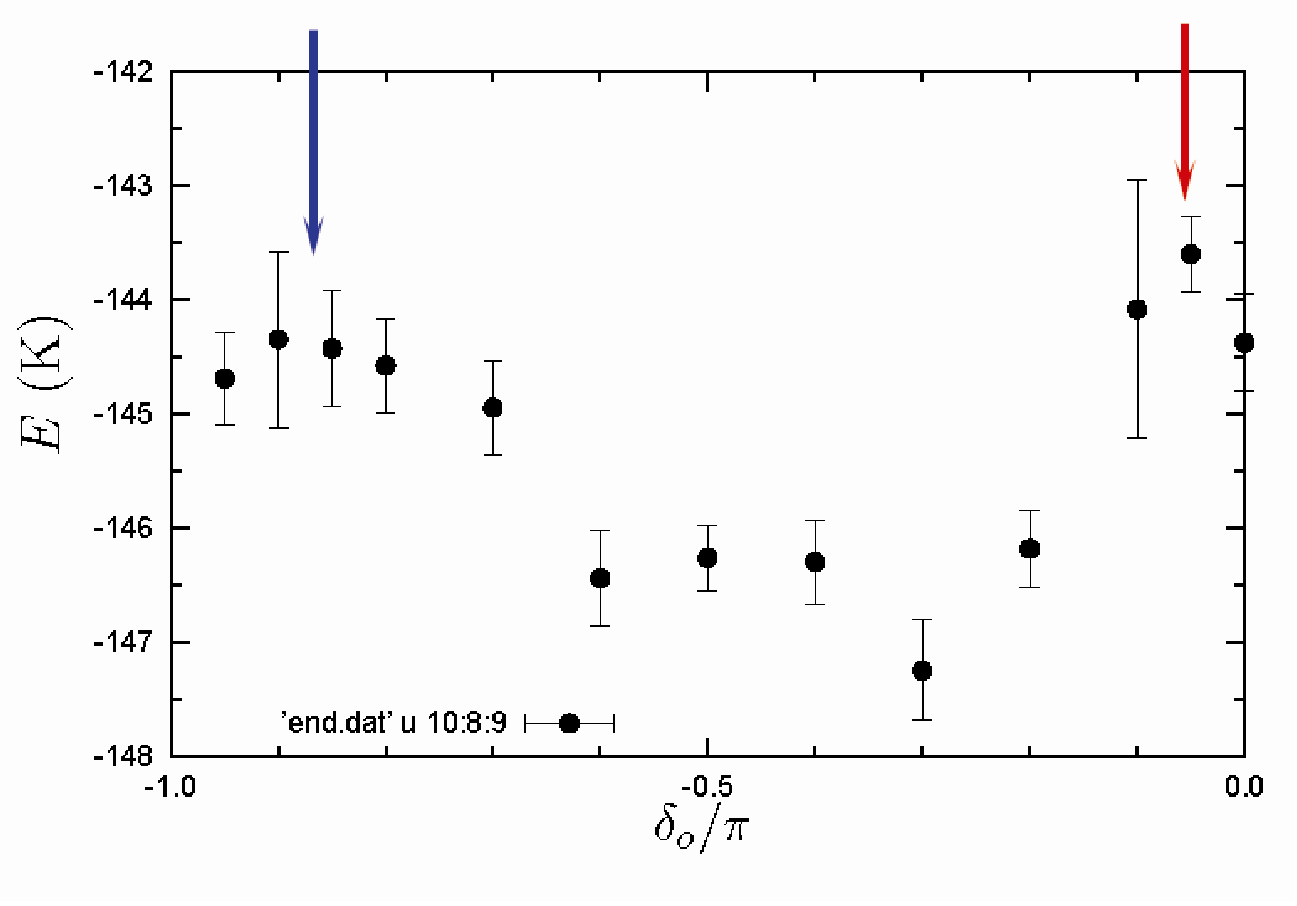
Scaled phase of the transmission coefficient vs. the wavevector of the incoming particle.

Phase results



Scaled phase of the transmission coefficient vs. the wavevector of the incoming particle.

Double peak structure



Energy vs. scattering phase for odd simulation with $k = 0.532\text{Å}$

Summary and future work

We can use DMC with fixed nodes approximation to simulate the transmission process by determining the scattering states for the problem.

Current results are strongly supportive of the condensate mediated transmission hypothesis.

Should eventually be able to compare different transmission channels.