SU(N) Magnetism with Cold Atoms and Chiral Spin Liquids

Victor Gurarie

collaboration with M. Hermele, A.M. Rey

UPC, Barcelona, July 5 2010
In this talk

- Alkaline earth atoms can be thought of as having SU(N) spins, generalization of the usual SU(2) spins.

- SU(N) magnets are more controllable theoretically than their SU(2) counterparts and have richer phase diagrams.

- “Heisenberg antiferromagnets” of the SU(N) spins can be Chiral Spin Liquids, spins counterparts of quantum Hall effect, states of matter having excitations with fractional and non-Abelian statistics. Those, as is well known, can be used for quantum computation.
SU(N) magnetism
Quantum magnetism

Cuprates’ phase diagram

\[ T \]

\[ \text{doping} \]

AF

SC

Antiferromagnet

(occurs when one has close to one electron per lattice site)
Quantum magnetism

Cuprates’ phase diagram

Origin of the antiferromagnetism: Mott insulator of spinful electrons

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\[ \hat{H} = -J \sum_{<ij> ; \alpha, \beta = \uparrow, \downarrow} \hat{f}_{i,\beta} \hat{f}_{i,\alpha} \hat{f}_{j,\beta} \hat{f}_{j,\alpha} \]

Antiferromagnet (occurs when one has close to one electron per lattice site)

spin-up  spin-down
Quantum magnetism

Cuprates’ phase diagram

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\[ \hat{H} = J \sum_{\langle ij \rangle; \alpha, \beta = \uparrow, \downarrow} \hat{f}_{i, \alpha}^{\dagger} \hat{f}_{i, \beta} \hat{f}_{j, \beta}^{\dagger} \hat{f}_{j, \alpha} \]

\[ \hat{S}_\alpha^\beta(i) \quad \hat{S}_\beta^\alpha(j) \]

\[ \hat{f}_{i, \uparrow}^{\dagger}, \hat{f}_{i, \uparrow} ; \hat{f}_{i, \downarrow}^{\dagger}, \hat{f}_{i, \downarrow} \]

spin-up, spin-down

Spin operators
Quantum magnetism

Cuprates’ phase diagram

Antiferromagnet
(occurs when one has close to one electron per lattice site)

\[
\hat{H} = -J \sum_{<ij>; \alpha, \beta = \uparrow, \downarrow} \hat{f}^\dagger_{j, \beta} \hat{f}_{i, \beta} \hat{f}^\dagger_{i, \alpha} \hat{f}_{j, \alpha}
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\]

\[
\hat{H} = J \sum_{<ij>} \hat{S}_i \cdot \hat{S}_j
\]

Origin of the antiferromagnetism:
Mott insulator of spinful electrons

\frac{\text{AF}}{\text{SC}}

\text{doping}

spin-up spin-down

\text{Spin operators}

Heisenberg antiferromagnet
Alkaline earth atoms: SU(N) spins

two electrons in the outer shell

Ground state $^1S_0$

Excited state $^3P_0$

Both of these states have $J=0$, so the nuclear spin is decoupled from the electronic spin.
Alkaline earth atoms: SU(N) spins

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This (sometimes large) nuclear spin can play the role of the SU(N) spin.

For example, $^{87}\text{Sr}$: $I=9/2$, $N=2I+1=10$.


M. Cazallila, A. Ho, M. Ueda (2009)
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\[ S \uparrow \downarrow \]

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Interesting twist: only fermionic atoms have $N>1$

(bosons have even-even nuclei, whose $I=0$)
Mott insulators of the alkaline earth atoms

$^{87}\text{Sr}$ atoms

nuclear spins
Mott insulators of the alkaline earth atoms

nuclear spins

$^{87}\text{Sr}$ atoms
Mott insulators of the alkaline earth atoms

$\text{Sr atoms}$

Creates an atom on a site $i$, with nuclear spin $\alpha$.

$f^\dagger_{i,\alpha}$
Mott insulators of the alkaline earth atoms

\[ \hat{H} = J \sum_{\langle ij \rangle, \alpha, \beta=1, \ldots, N} \hat{f}^{\dagger}_{i,\alpha} \hat{f}_{i,\beta} \hat{f}^{\dagger}_{j,\beta} \hat{f}_{j,\alpha} \]

Creates an atom on a site \( i \), with nuclear spin \( \alpha \).

\[ \hat{f}^{\dagger}_{i,\alpha} \]

\( ^{87}\text{Sr} \) atoms

nuclear spins

SU(N) spins
Mott insulators of the alkaline earth atoms

$^{87}$Sr atoms

\[ \hat{f}_{i,\alpha} \to \sum_{\beta=1}^{N} U_{\alpha,\beta} \hat{f}_{i,\beta} \]

SU(N) symmetry

\[ \hat{H} = J \sum_{<ij>, \alpha,\beta=1,...,N} \hat{f}^\dagger_{i,\alpha} \hat{f}_{i,\beta} \hat{f}^\dagger_{j,\beta} \hat{f}_{j,\alpha} \]

SU(N) spins

\[ \hat{S}^\alpha_{\beta}(i) \hat{S}^\beta_{\alpha}(j) \]

Bottom line: these are SU(N) spin antiferromagnets
Experiments with $^{87}\text{Sr}$, SU(10)

Degenerate Fermi Gas of $^{87}\text{Sr}$

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian
Rice University, Department of Physics and Astronomy, Houston, Texas, 77251

(Dated: May 6, 2010)

We report quantum degeneracy in a gas of ultra-cold fermionic $^{87}\text{Sr}$ atoms. By evaporatively cooling a mixture of spin states in an optical dipole trap for 10.5 s, we obtain samples well into the degenerate regime with $T/T_F = 0.26^{+0.05}_{-0.06}$. The main signature of degeneracy is a change in the momentum distribution as measured by time-of-flight imaging, and we also observe a decrease in evaporation efficiency below $T/T_F \sim 0.5$.

Double-degenerate Bose-Fermi mixture of strontium

Meng Khoon Tey, 1 Simon Stellmer, 1, 2 Rudolf Grimm, 1, 2 and Florian Schreck 1

1 Institut für Quantenoptik und Quanteninformation (IQOQI), Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria
2 Institut für Experimentalphysik und Zentrum für Quantenphysik, Universität Innsbruck, 6020 Innsbruck, Austria

(Dated: June 8, 2010)

We report on the attainment of a spin-polarized Fermi sea of $^{87}\text{Sr}$ in thermal contact with a Bose-Einstein condensate (BEC) of $^{84}\text{Sr}$. Interisotope collisions thermalize the fermions with the bosons during evaporative cooling. A degeneracy of $T/T_F = 0.30(5)$ is reached with $2 \times 10^4$ $^{87}\text{Sr}$ atoms together with an almost pure $^{84}\text{Sr}$ BEC of $10^5$ atoms.
Experiments with $^{173}\text{Yb}$, SU(6)

Realization of SU(2) × SU(6) Fermi System

Shintaro Taie, Yosuke Takasu, Seiji Sugawa, Rekishu Yamazaki, Takuya Tsujimoto, Ryo Murakami, and Yoshiro Takahashi

1 Department of Physics, Graduate School of Science, Kyoto University, Japan 606-8502
2 CREST, JST, 4-1-8 Honcho Kawaguchi, Saitama 332-0012, Japan

(Dated: May 21, 2010)

We report the realization of a novel degenerate Fermi mixture with an SU(2) × SU(6) symmetry in a cold atomic gas. We successfully cool the mixture of the two fermionic isotopes of ytterbium $^{171}\text{Yb}$ with the nuclear spin $I = 1/2$ and $^{173}\text{Yb}$ with $I = 5/2$ below the Fermi temperature $T_F$ as $0.46T_F$ for $^{171}\text{Yb}$ and $0.54T_F$ for $^{173}\text{Yb}$. The same scattering lengths for different spin components make this mixture featured with the novel SU(2) × SU(6) symmetry. The nuclear spin components are separately imaged by exploiting an optical Stern-Gerlach effect. In addition, the mixture is loaded into a 3D optical lattice to implement the SU(2) × SU(6) Hubbard model. This mixture will open the door to the study of novel quantum phases such as a spinor Bardeen-Cooper-Schrieffer-like fermionic superfluid.

PACS numbers: 03.75.Ss, 67.85.Lm, 37.10.Jk
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Shin’Aro Taie,1,∗ Yosuke Takaoka,1 Seiji Sugawa,1 Rekishu Yamazaki,1,2 Takuya Tsujimoto,1 Ryo Murakami,1 and Yoshiro Takahashi1,2

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FIG. 5. (Color online) Quasimomentum distribution of (a) $^{171}$Yb and (b) $^{173}$Yb in the SU(2)×SU(6) two-species mixture in an optical lattice. The density distributions integrated along the vertical direction are also shown below. The atom numbers are $0.4 \times 10^4$ for $^{171}$Yb and $1.5 \times 10^4$ for $^{173}$Yb, respectively. The images are taken after linear ramping down of the lattice in 0.5ms, followed by a ballistic expansion of (a) 12ms and (b) 13ms. The dotted lines indicate the domain of the 1st Brillouin zone, which equals twice the recoil momentum $\hbar k$. 
**SU(N) antiferromagnets**

SU(N) spins are analytically tractable in the large N limit. There is a long history of studying SU(N) spin antiferromagnets, to better understand the usual SU(2) spin antiferromagnets.
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Late 1980’s

Careful and numerous studies of the SU(N) antiferromagnets. Papers with many 100’s of citations.

S. Sachdev  N. Read  I. Affleck  J. B. Marston
SU(N) antiferromagnets

SU(N) spins are analytically tractable in the large N limit. There is a long history of studying SU(N) spin antiferromagnets, to better understand the usual SU(2) spin antiferromagnets.

Does this mean we can just look up the answer in these papers and find out all we need to know about SU(N) antiferromagnets and alkaline earth atoms?

NO!
A collection of spin-1/2s on a square lattice in the presence of the antiferromagnetic interactions at T=0 forms a Néel state with a long range antiferromagnetic order (this is known numerically and experimentally).
The difference between SU(2) and SU(N)

SU(2) spins: two spins-1/2 can form a singlet
The difference between SU(2) and SU(N)

SU(2) spins: two spins-1/2 can form a singlet

\[ \phi^{(1)}_{\uparrow} \phi^{(2)}_{\downarrow} - \phi^{(1)}_{\downarrow} \phi^{(2)}_{\uparrow} \]
	singlet
The difference between SU(2) and SU(N)

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SU(N) spins: at least N spins needed to form a singlet

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SU(2) spins: two spins-1/2 can form a singlet

\[ \phi^{(1)}_{\uparrow} \phi^{(2)}_{\downarrow} - \phi^{(1)}_{\downarrow} \phi^{(2)}_{\uparrow} \]

This is not a singlet, but an antisymmetric \( N \) by \( N \) tensor, with \( N(N-1)/2 \) components.

SU(N) spins: at least \( N \) spins needed to form a singlet

2 spins:

\[ \phi^{(1,2)}_{\alpha \beta} = \phi^{(1)}_{\alpha} \phi^{(2)}_{\beta} - \phi^{(1)}_{\beta} \phi^{(2)}_{\alpha} \]

\( \alpha, \beta = 1, 2, \ldots, N \)
The difference between SU(2) and SU(N)

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\[ \phi_{\alpha\beta}^{(1,2)} = \phi^{(1)}_{\alpha} \phi^{(2)}_{\beta} - \phi^{(1)}_{\beta} \phi^{(2)}_{\alpha} \]

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3 spins:

\[ \phi^{(1,2,3)}_{\alpha\beta\gamma} \]

antisymmetric rank 3 tensor: \( N(N-1)(N-2)/3! \) components
The difference between SU(2) and SU(N)

SU(2) spins: two spins-1/2 can form a singlet

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antisymmetric rank 3 tensor: \( N(N-1)(N-2)/3! \) components

\( N \) spins:

\[ \phi_{\alpha_1 \alpha_2 \ldots \alpha_N}^{(1,2,\ldots,N)} \]

finally, scalar!
Prior studies of the SU(N) magnets

All prior studies were designed so that one was able to form singlets from nearby spins.
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Method: Place $m$ and $N-m$ spins on even and odd sublattices respectively.

1. Read & Sachdev, 1989: $m=1$
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Analytics at large \(N\): The ground state is Valence Bond Solid (VBS)
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Numerics: the ground state is

- Néel if $N<4$
- VBS if $N>4$

Analytics at large $N$: The ground state is Valence Bond Solid (VBS)
All of this is not relevant for us: we can place one, at most two atoms (SU(N) spins) on each site.

Thus this is a new yet unexplored problem.
One atom (or two) per site: experimentally realizable SU(N)

at least $N$ (or $N/2$) sites are required to form a singlet

D. Arovas (2008) calls these N-simplexes.
Large $N$ methods
Large N methods

Taking \( N \) to infinity is problematic: the number of spins required to form a singlet goes to infinity too.

Proposal: Let us put \( m = \frac{N}{k} \) atoms on each site.

The number of sites required to form a singlet is now \( k \) and is \( N \) independent. Then take \( N \) to infinity.

Carrying out this procedure results in the chiral spin liquid ground state if \( k > 4 \).

M. Hermele, VG, A.-M. Rey (2009)
Saddle point approximation

\[ H = -\frac{J}{N} \sum_{\langle i,j \rangle, \alpha, \beta = 1, \ldots, N} \hat{f}_{j,\beta} \hat{f}_{i,\beta} \hat{f}_{i,\alpha} \hat{f}_{j,\alpha} \]

convenient for large N limit
Saddle point approximation

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convenient for large N limit

Hubbard-Stratonovich hopping

\[ S = \int d\tau \left[ \sum_i \left\{ \sum_{\alpha} \bar{f}_{i, \alpha} \partial_\tau f_{i, \alpha} + i\lambda_i \left( \sum_{\alpha} \bar{f}_{i, \alpha} f_{i, \alpha} - \frac{N}{k} \right) \right\} + \sum_{\langle ij \rangle} \left\{ \chi_{ij} \sum_{\alpha} \bar{f}_{i, \alpha} f_{j, \alpha} + \text{H.c.} + N \frac{\chi_{ij}^2}{J} \right\} \right] \]
Saddle point approximation

\[ H = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha, \beta = 1, \ldots, N} \hat{f}^\dagger_{j,\beta} \hat{f}_{i,\beta} \hat{f}^\dagger_{i,\alpha} \hat{f}_{j,\alpha} \] convenient for large N limit

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Saddle point equations (exact in the large N limit)

\[ \frac{1}{k} = \left\langle \hat{f}^\dagger_{i,\alpha} \hat{f}_{i,\alpha} \right\rangle \quad \chi_{ij} = -J \left\langle \hat{f}^\dagger_{j,\alpha} \hat{f}_{i,\alpha} \right\rangle \]

Fermions try to arrange their hopping dynamically to minimize their energy
Saddle point approximation

\[ H = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha, \beta = 1, \ldots, N} \hat{f}_{i, \beta} \hat{f}_{i, \beta} \hat{f}_{i, \alpha} \hat{f}_{j, \alpha} \]

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Saddle point equations (exact in the large N limit)

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\[
\chi_{ij} = \chi_{ij}^{(0)} e^{iA_{ij}}
\]

Fermions try to arrange their hopping dynamically to minimize their energy
Saddle points for $k=2, 3, 4$
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Spins pair up to form 2-spin singlets

This state (VBS) was found by Affleck & Marston in 1989

Nonzero hopping $\chi^{(0)}$
Saddle points for $k=2, 3, 4$

- Nonzero hopping
- $\chi^{(0)}$

$k = 2$

Spins pair up to form 2-spin singlets
This state (VBS) was found by Affleck & Marston in 1989

$k = 3$

Spins form 6-spin singlets
(new result)
Saddle points for $k=2, 3, 4$

- Nonzero hopping $\chi^{(0)}$
  - $k=2$
    - Spins pair up to form 2-spin singlets
    - This state (VBS) was found by Affleck & Marston in 1989
  - $k=3$
    - Spins form 6-spin singlets
      (new result)
  - $k=4$
    - Spins form 4-spin singlets
      (new result)
Saddle points for \( k>4 \)

To minimize their energy, fermions attempt to organize hoppings so that they completely fill a band.

The filling fraction for fermions with one of \( N \) spin components is

\[
\nu = \frac{1}{N} \frac{N}{k} = \frac{1}{k}
\]

Fermions would like to form a closed band with \( N_s/k \) states.

Our result: the best way to do that is by arranging a “magnetic flux” of \( 2\pi/k \) per plaquette and fill the lowest Landau level.

M. Hermele, VG, A.-M. Rey (2009)
Chiral Spin Liquid
Chiral Spin Liquid (CSL)

Wen, Wilczek & Zee  Kalmeyer & Laughlin
Proposed in 1989

A state of magnets
without any magnetic order (spin liquid),
but breaking parity and time reversal invariance (chiral).

Has to be described by a Chern-Simons theory.

(Local) Hamiltonians whose ground state would be CSL
were unknown until now
Chiral spin liquid \((k>4)\)

\[
H = \sum_{<ij>,\alpha} \chi^{(0)}_{ij} \left( \hat{f}^{\dagger}_{i,\alpha} \hat{f}_{j,\alpha} + \text{H.c.} \right)
\]

magnetic field with \(1/k\) flux through plaquette

like quantum Hall effect

\[\chi_{ij} = \chi^{(0)}_{ij} e^{iA_{ij}}\]

\[
S_{\text{eff}} = \frac{N}{4\pi} \int d^2x dt \epsilon_{\mu\nu\rho} A_\mu \partial_\nu A_\rho
\]

Level \(N\) Chern-Simons theory

Fermions acquire fractional statistics with the angle \(\theta\):

\[
\theta = \pi + \frac{\pi}{N} \quad N = k, 2k, 3k, \ldots
\]
Chiral spin liquid \((k>4)\)

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Chiral spin liquid ($k > 4$)

$$H = \sum_{<ij>, \alpha} \chi_{ij}^{(0)} \left( \hat{f}_{i, \alpha}^\dagger \hat{f}_{j, \alpha} + \text{H.c.} \right)$$

magnetic field with $1/k$ flux through plaquette

like quantum Hall effect

$$\psi(\mathbf{r}_2, \mathbf{r}_1) = e^{i(\pi + \frac{\pi}{N})} \psi(\mathbf{r}_1, \mathbf{r}_2)$$

$$\chi_{ij} = \chi_{ij}^{(0)} e^{iA_{ij}}$$

$$S_{\text{eff}} = \frac{N}{4\pi} \int d^2x dt \epsilon_{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$  Level $N$ Chern-Simons theory

Fermions acquire fractional statistics with the angle $\theta$:

$$\theta = \pi + \frac{\pi}{N} \quad N = k, 2k, 3k, \ldots$$
Who’s the carrier of the statistics?

\[ \hat{S}^{\alpha}_{\beta}(i) = \hat{f}^\dagger_{i,\alpha} \hat{f}_{i,\beta} \]

The spin itself is not fractional

The fermions are fractional, but with each site containing exactly one atom, the fermionic atoms don’t have any dynamics, fractional or otherwise.

Let’s create “holes” - empty atomless sites on the lattice:

\[ \hat{f}_{i,\alpha} = \hat{b}^\dagger_{i} \hat{c}_{i,\alpha} \]

“spinon” - fermion carrying spin

“holon” - boson carrying atom number

Both spinons and holons are fractional, but only holons respond to an external potential.

Well known construction from high T_c theories - only here justified by large N.

P.A. Lee, N. Nagaosa (1992)
Scenario to create fractional excitations

Lowering the potential at one site localizes a fractional particle at that site.

Localized fractional holon
Scenario to create fractional excitations

Lowering the potential at one site localizes a fractional particle at that site.

site potential

localized fractional holon
Non-Abelian chiral spin liquid

Place two species of fermionic atoms on each site, in two distinct electronic states $^1S_0$ and $^3P_0$.

They form either antisymmetric (boring) or symmetric (interesting) state depending on the relative strength of interaction constants (10 author paper)

\[ a, b = ^1S_0, ^3P_0 \] labels species

\[ |s\rangle = \frac{1}{\sqrt{2}} \left( \hat{f}_{a,\alpha}^\dagger \hat{f}_{b,\beta}^\dagger + \hat{f}_{b,\alpha}^\dagger \hat{f}_{a,\beta}^\dagger \right) |0\rangle \] on every site of the lattice

\[ H = \sum_{\langle ij\rangle, a, b, \alpha, \beta} \hat{f}_{i,a,\alpha}^\dagger \hat{f}_{i,a,\beta} \hat{f}_{j,b,\beta}^\dagger \hat{f}_{j,b,\alpha} \]
Non-Abelian chiral spin liquid

Place two species of fermionic atoms on each site, in two distinct electronic states $^1S_0$ and $^3P_0$.

$$H = \sum_{i,j,a,b,\alpha} \chi_{ij}^{(0)ab} \hat{f}^\dagger_{i,a,\alpha} \hat{f}_{j,b,\alpha}$$

$$\chi_{ij}^{ab} = \chi_{ij}^{(0)ab} e^{iA_{ij}^{ab}}$$

$a, b = {}^1S_0, {}^3P_0$ labels species

$$\chi_{ij}^{(0)ab} = \delta_{ab} \chi_{ij}^{(0)}$$

magnetic field

$$S_{CS} = \frac{N}{4\pi} \text{Tr} \int d^2x dt \, \epsilon_{\mu\nu\rho} \left[ A_{\mu} \partial_{\nu} A_{\rho} + \frac{2}{3} A_{\mu} A_{\nu} A_{\rho} \right]$$

This is non-Abelian Chern-Simons SU(2)$_N$ theory!
Non-Abelian chiral spin liquid

Place two species of fermionic atoms on each site, in two distinct electronic states $^1S_0$ and $^3P_0$.

$$H = \sum_{i,j,a,b,\alpha} \chi_{ij}^{(0)ab} \hat{f}_{i,a,\alpha} \hat{f}_{j,b,\alpha} \chi_{ij}^{ab} = \chi_{ij}^{(0)ab} e^{i A_{ij}^{ab}}$$

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This is non-Abelian Chern-Simons SU(2)_N theory!

Topological quantum computing with SU(2)_10!
Non-Abelian chiral spin liquid
Topological quantum computing

Non-Abelian particles are perfect qubits

A. Kitaev, 1997
Non-Abelian particles are perfect qubits

\[ \psi_\alpha(r_1, r_2) \]

A. Kitaev, 1997
Non-Abelian particles are perfect qubits

\[ \psi_\alpha(r_2, r_1) = \sum_\beta U_{\alpha, \beta} \psi_\beta(r_1, r_2) \]
Topological quantum computing
Topological quantum computing

We wish we had an SU(2)_2 Pfaffian, p+ip superconductor, all those Majorana fermions...

How about SU(2)_{10}!
Phase diagram (Abelian)

\[ N = km \]

- VBS
- CSL
- Néel

# of atoms needed for a singlet

\( k \)

\( m \)

# of atoms per site
$N = km$

Phase diagram (Abelian)
Phase diagram (Abelian)

Potential experiment lives here

Néel

VBS

$N = km$

# of atoms

$\kappa$

# of atoms needed for a singlet

topological quantum computing

Potential experiment lives here

CSL

$N = km$

# of atoms per site
Phase diagram (Abelian)

Potential experiment lives here

\[ N = k m \]
Phase diagram (Abelian)

Potential experiment lives here

VBS?

Néel

VBS

\[ N = km \]
Status of numerics

• Numerical methods are the only way to access the experimentally relevant $m=1$ column

• QMC - sign problem for $k>2$, at least in 2D

• Other 2D numerical methods?
Conclusions

‣ SU(N) magnets are a useful theoretical construct due to the existence of the large $N$ techniques.

‣ SU(N) magnets can have phases going beyond the phases of the SU(2) magnets.

‣ Nuclear spin of the alkaline earth atoms a perfect realization of the SU(N) spin - so far lacking in condensed matter.

‣ A version of the SU(N) magnets particularly well suited to realization by the alkaline earth atoms forms chiral spin liquids, a state of matter with fractionalized excitations.

‣ Possibility of the topological quantum computing with the SU(N) spin magnets??
The end